THE ELEMENTS OF STRUCTURES

Published by the
McGraw-Hill Book Company
New York

Successors to the Book Departments of the
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ENGINEERING EDUCATION SERIES

THE ELEMENTS OF STRUCTURES

PREPARED IN THE
EXTENSION DIVISION OF
THE UNIVERSITY OF WISCONSIN

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FIRST EDITION

McGRAW-HILL BOOK COMPANY
239 WEST 39TH STREET, NEW YORK
6 BOUVERIE STREET, LONDON, E. C.
1912

Printed and Electrotyped
by The Maple Press
York, Pa.

PREFACE

This text has been developed solely with the idea of using it in correspondence-study. It has been the aim of the writer to deal with the fundamentals in structural engineering in a simple and interesting manner and, as far as possible, to make the text speak to the student with a directness similar to that employed by an instructor to a student in residence. In order to obtain these results, some of the usual forms of text-book presentation have been disregarded.

The work is intended for students who have had an ordinary training in Arithmetic, Algebra, Plane and Solid Geometry, Logarithms, Trigonometry, Mechanical Drawing, and Strength of Materials. It is a prerequisite course to all the regular structural engineering studies offered by the Extension Division of The University of Wisconsin and treats only of the general methods to be followed in design. The methods in detail for the different types of structures are given in the courses which follow.

The author is indebted to Mr. W. S. Kinne, Assistant Professor of Structural Engineering in The University of Wisconsin, for a careful reading of the proof and for many valuable suggestions and criticisms.

G. A. H.

THE UNIVERSITY OF WISCONSIN
MADISON, WISCONSIN,
January 1, 1912.

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THE ELEMENTS OF STRUCTURES

ASSIGNMENT 1

CHAPTER I

GENERAL CONSIDERATIONS

1. Definitions.—In this course a structure will mean a part, or an assemblage of parts, constructed so as to support certain known loads. Structures are acted upon by external forces and these external forces are held in equilibrium by internal forces, called *stresses.*

A girder is a beam usually of large proportions which carries other beams. It may be a single piece or made up of a number of pieces. In a framed floor it is one of the main members, the girders supporting the cross beams which in turn carry the flooring.

A truss is a framed or jointed structure. It is composed of straight members which are connected only at their intersections, so that if the loads are applied at these intersections the stress in each member is in the direction of its length; that is, for loads so applied, each member is subjected only to a direct tensile or a direct compressive stress.

A simple structure is one which is supported at its ends and which under vertical loads brings only vertical pressures on the supports. Simple structures are said to be *simply supported.*

A roof truss is a structure designed to support the weight of a certain area of roof as well as the proportional part of such other loads as may properly be transmitted to it. The ends of the truss may be supported upon side walls or upon columns. The *span* is the horizontal distance in feet between the centers of supports. The *rise* is the distance from the highest point of the truss to the line joining the points of support. The *pitch* is

the ratio of the rise of the truss to its span. The *upper chord* consists of the upper line of members. The *lower chord* consists of the lower line of members. The *web members* connect the joints of the upper chord with those of the lower chord. (Fig. 1.)

A bridge truss is one of the main parts of a bridge and is designed to support the weight of a certain area of bridge floor as well as the proportional part of such other loads as may properly be transmitted to it. Plate girders are often used in place of trusses. The bridge trusses (or girders as the case may be) and the floor, together with whatever bracing is used, comprise the *superstructure* of a bridge. The *substructure* of a bridge includes that part of the bridge upon which the superstructure rests and may consist of masonry piers and abutments, or of

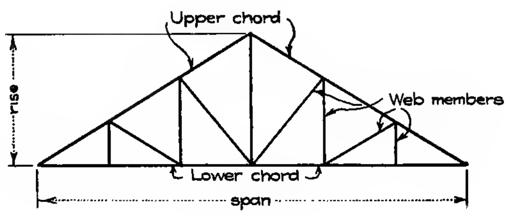

FIG. 1.

steel towers. A *bridge* thus includes both the substructure and superstructure, but bridges are generally classified without any reference to the substructure. A *deck* bridge is one in which the floor is at the top of the superstructure; that is, the floor is supported at or near the top chord of the trusses or girders. A *through* bridge is one in which the greater part of the superstructure is above the floor level. Except in the case of a pony truss (one in which there is no overhead bracing), a system of overhead lateral bracing is used with all through truss bridges. Each bridge truss generally consists of a top chord, a bottom chord, end posts, and web members. In the trusses shown in Figs. 5 and 6 the top and bottom chords and the end posts are designated, while the remaining parts of the trusses are the web members.

2. Classification of Engineering Structures.—Engineering structures may be classified according to material. Only the principal structures need be mentioned.

Structures of stone, brick, or concrete, include retaining walls,

dams, bridge abutments, bridge piers, culverts, arches, and walls of buildings.

Structures of wood include beams, roofs, pile and frame trestles, lattice and truss bridges, and the frames of buildings.

Structures of iron or steel include beams, girders, floors, columns, roof trusses, and various types of bridges.

Combination structures include combination timber and steel bridges, and all types of reinforced concrete structures.

3. Outer Forces.—The external or outer forces acting upon a structure comprise the loads which the structure is designed to carry and also the pressures brought to bear at the points of support. These pressures are called reactions and are forces brought into action at the points where a structure is supported in order that the structure may stand.

4. Inner Forces.—The internal or inner forces are the stresses in the different members of a structure which are called into action by the outer forces and by means of which the structure carries its load.

CHAPTER II

LOADS ON STRUCTURES

5. Dead and Live Loads.—A structure is designed to carry both dead and live loads. The dead load is the weight of the structure itself and permanent loads, if any. The live load is any moving or variable load (or loads) which may come upon the structure.

In the case of roof trusses, the weight of snow and the pressure of wind are the live loads. In some cases the live load on roof trusses comprises also accidental loads, as, for example, where a system of pulleys is attached to the roof by means of which heavy weights are lifted.

In a highway bridge, not carrying electric car tracks, the live loads are the weight of a crowd of people or of heavy wagons, the weight of snow, and the pressure of wind.

In a railroad bridge, the live load consists of locomotives and cars, the weight of snow, and the pressure of wind. (Centrifugal force is produced by the lateral effect of moving loads on bridges built on a curve. This force produces stress in the members like any other live load and is considered as such. The nature

of this force will be explained in "Bridge Trusses, Part 1," Course 412. The application of brakes on a railroad train and the impact, or dynamic action, of the live load also produce stress in the members of a bridge, but they will be given no further consideration here).

In the floors and walls of buildings the live load may come from a crowd of people or from the weight of heavy material.

The live load on other structures might be enumerated, but the above should be sufficient to make it clear to the student the distinction between the dead and live loads on any structure.

6. Weight of Structures.—A structure should be designed to carry any loads that are likely to come upon it. In order that a structure may do this, the members must be of such size that they will resist the stresses which will be produced in them by the application of both the dead and live loads. But the size of the members of a structure affects the weight of the structure, which is either a part or the whole of the dead load. Consequently, since the members of a structure are to be proportioned for the live load plus the dead load, it may be seen how impossible it is to accurately determine the weight of a proposed structure before starting to design it.

The weight, however, may always be found by successive approximation; that is, by first assuming it and then proportioning the structure for this assumed weight, plus any permanent load, plus the known live loads; after which the weight may be found more accurately, and if necessary, the computations revised accordingly.

There are some structures, such as arches and other masonry structures, the complete size and shape of which is assumed at the start and then it is determined whether the structure, as designed, is sufficient for the purpose for which it is to be built. Such structures will not be considered in this course.

7. Formulas for Computing Weight.—Empirical formulas for computing weight have been determined for some of the principal structures. They are generally derived from the records of actual practice and may be employed under ordinary conditions to obtain approximate values which can be used in preliminary design. The more familiar structures, of which the approximate weight may be computed by the use of formulas, are roof trusses and bridges.

8. Dead Load on Roof Trusses.—The dead load on a roof

truss consists of the weight of the truss itself and the weight of the roof it supports.

The weight of the truss depends upon the span, the distance apart of the adjacent trusses, the weight of the roof covering and other elements of design. Conditions vary so greatly that empirical formulas for weight cannot be expected to cover trusses of a special type or with special loading. In such cases the designer must rely upon his judgment to a more or less degree in selecting the trial weight.

The approximate weight of ordinary types of wooden and steel roof trusses may be found by the following formula given by Merriman and Jacoby:

$$w = C\left(1 + \frac{L}{10}\right)$$

in which w = weight of truss in pounds per square foot of horizontal projection of the roof supported.

C = constant; for wood 0.50, for steel 0.75.
L = span of the truss in feet.

From an investigation of trusses designed for the purpose, Professor N. Clifford Ricker has deduced the formula [1]

$$w = \frac{L}{25} + \frac{L^2}{6000}$$

for a common type of wooden roof truss, in which w and L have the same meaning as in the preceding formula. The span lengths used varied from 20 to 200 ft. and the height of truss in each case was taken as one-quarter of the span length. Although this formula was derived mainly with reference to one particular type of truss it has, however, been found to give approximate values for the common types of roof trusses of both wood and steel.

The position of roof trusses in a building is shown in Fig. 2. Each truss sustains the weight of a certain area of roof. This area for a given truss is included between two sections which are located on opposite sides of the given truss and midway between the given truss and the truss adjacent. Roof truss

[1] See "Bulletin No. 16" of the University of Illinois Engineering Experiment Station, August, 1907.

"*A*," for example, in Fig. 2, supports the weight of the roof between the section *AA* and the section *BB*. Roof truss "*B*" supports the weight of the roof between the section *BB* and the

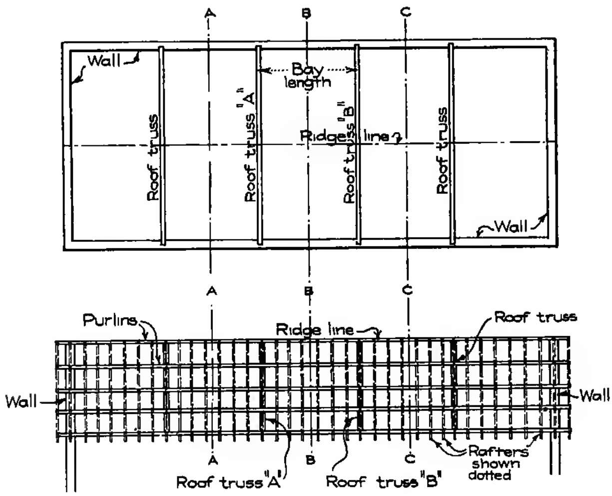

FIG. 2.

section *CC*. This area is the same as that included between adjacent trusses, when the trusses are equally spaced. The space between two adjacent trusses is called a *bay*, and the *bay length* is the distance from center to center of trusses. (Fig. 2.) A

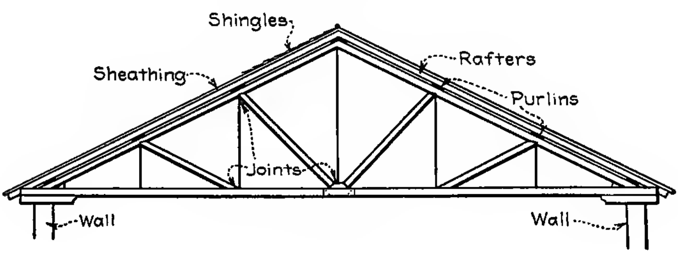

FIG. 3a.

truss may be so placed, for example, at the end of a building, that it supports the roof only to one side of it.

The roof consists of a covering of tin, slate, tiles, wooden shingles or whatever the material may be, resting usually upon a sheathing

of plank, and these in turn are supported by purlins, which are beams of wood or steel, extending from truss to truss. In many cases the sheathing of plank rests on members called jack-rafters (or common rafters), usually of wood, which extend

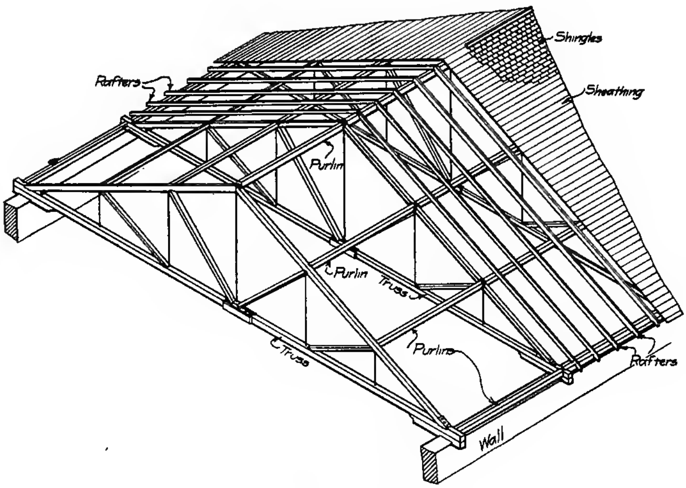

FIG. 3b.

parallel to the main trusses and these in turn rest upon the purlins. (Figs. 2, 3a, 3b, 3c, and 3d). Sheathing is commonly employed in connection with roof coverings, but it is often omitted when corrugated iron is used.

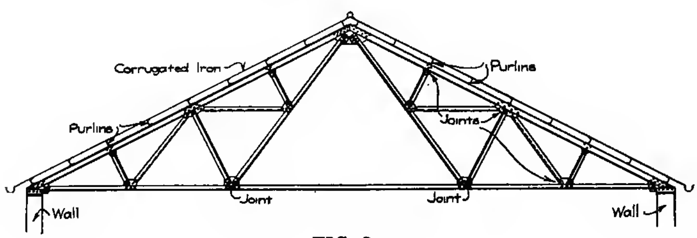

FIG. 3c.

The weight of roof covering varies greatly, depending upon the materials employed. Approximate weights per square foot of roof surface are—tin, 1 lb.; wooden shingles, 2 to 3 lb.; slates, 8 to 10 lb.; tiles, 8 to 20 lb.; corrugated iron, 1 to 3 lb.; gravel and felt, 5 to 6 lb. Sheathing; boards 1 in. thick, 3 to 5 lb.

Rafters will weigh from 1.5 to 3 lb. per square foot of roof surface. Wooden purlins will weigh from 1.5 to 3 lb. and steel purlins from 1.5 to 4 lb. per square foot of roof surface.

Since the weight of the roof covering is supported by the purlins and these in turn by the trusses, the entire weight of an area of roof covering supported by a given truss is brought to bear upon the truss at the points where the purlins are fastened to the upper chord. (Figs. 3a, 3b, 3c, and 3d).

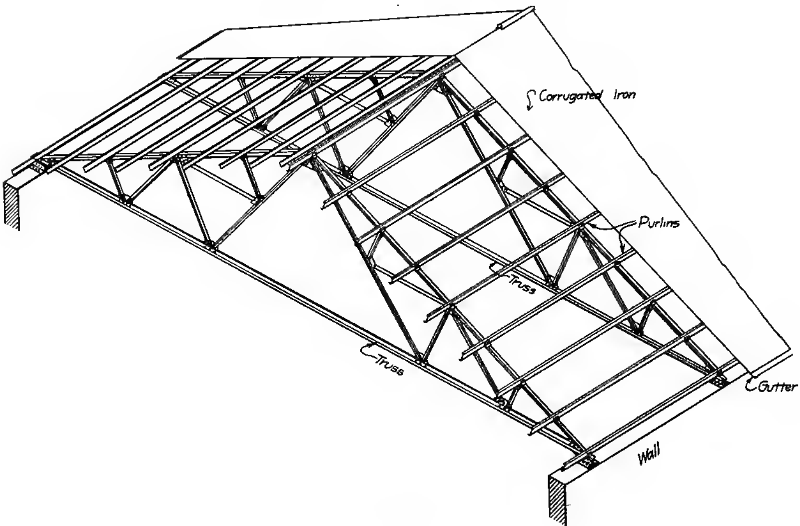

FIG. 3d.

In some cases it happens that ceilings are attached at the joint in the lower chord of roof trusses and when such is the case, we must, of course, account the proper load as resting on these joints. These loads are permanent and should be included in the total dead load. Plastered ceilings weigh about 10 lb. per square foot of ceiling.

A system of sway bracing is often used for rigidity. This bracing may be in the plane of the upper chord, in the plane of the lower chord, or in both planes. If used in both planes its weight will be from 0.5 to 1 lb. per square foot of roof surface.

The panel length of a roof truss is the distance between two successive joints of the upper chord measured along the slope, the purlins being fastened at these joints. These joints (such

as A, B, C, D, etc., Fig. 4) are called panel points. It often happens that purlins are placed at points on the upper chord other than at the joints, and in such a case the upper chord members are subjected to bending in addition to direct stress. (Fig. 3d).

The dead loads acting at the panel points are called dead panel loads. A dead panel load includes the load brought down at a panel point by the weight of the roof and also the weight which is said to be created at a panel point by the weight of the roof truss. It is not theoretically correct to consider the weight

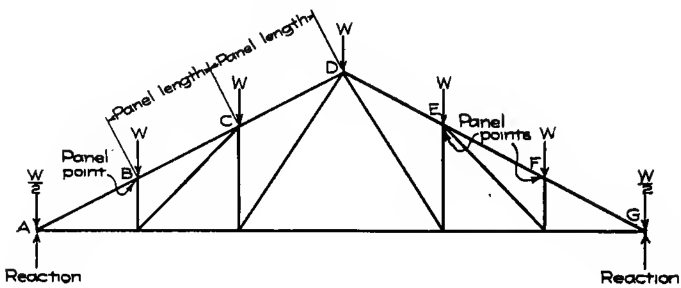

FIG. 4.

of the truss as concentrated at the upper joints, but it is customary and does not entail any appreciable error. For large trusses, however, a proper part of the load should be distributed over the bottom chord also.

The weight of a roof truss is generally expressed in terms of the horizontal projection of the roof supported. On the other hand, the weight of roof covering, sheathing, purlins, and also of the jack-rafters, if used, are expressed in pounds per square foot of roof surface.

Suppose that roof trusses, such as that shown in Fig. 4, are spaced 15 ft. center to center and that the entire weight of roof is 10 lb. per square foot of roof surface. In this roof truss the panel lengths AB, BC, CD, etc., are all equal, consequently the weight of roof which will act at panel points B, C, D, E, and F may be found by multiplying 15 by the panel length (which gives the area of roof surface supported at any one of these panel points) and then multiplying this result by 10. To the weight of roof acting at a panel point should be added

a panel weight of the roof truss in order to obtain the dead panel load.

The dead panel load on points A and G will be only one-half the above amount, since the purlins crossing at these points carry only one-half as much roof area as the purlins at the other panel points. The dead loads on roof trusses always act vertically according to the law of gravitation.

In special cases of unsymmetrical trusses, the dead load at each panel point must be found separately, due to the different areas of roof supported at these points. The weight of the roof truss must also be correctly proportioned among the different panel points.

Illustrative Problem.—Determine the approximate dead panel load on panel point B of the steel roof truss shown in Fig. 4 under the following conditions:—span, 55 ft.; rise, 14 ft.; bay length, 15 ft. The weight of the roof will be taken as 10.5 lb. per sq. ft. of roof surface made up of the following items:—shingles, 2.5 lb.; sheathing, 3.0 lb.; rafters, 2.0 lb.; purlins and bracing, 3.0 lb.

$$w=C\left(1+\frac{L}{10}\right)$$

$$=(0.75)\left(1+\frac{55}{10}\right)$$

$=4.9$ lb., weight of truss in lb. per sq. ft. of horizontal area.

Length of horizontal projection of an upper chord panel will be

$$\frac{55}{6}=9.2 \text{ ft.}$$

As the horizontal projection per panel is 9.2 ft. and the distance between trusses is 15 ft., the panel load due to the weight of the truss will be

$$(9.2)(15)(4.9)=676 \text{ lb.}$$

$$2AD=\sqrt{\overline{28}^2+\overline{55}^2}=61.7 \text{ ft.} \quad \frac{61.7}{6}=10.3 \text{ ft. (panel length).}$$

The panel load due to the roof will be

$$(10.3)(15)(10.5)=1622 \text{ lb.}$$

The total dead panel load at B will be

$676+1622=2298$ lb., or say 2300 lb. *Answer.*

Note.—It will be more convenient in the work which follows (although not compulsory) for the student to make many of the necessary computations with the aid of a slide rule. A slide rule should also be used in all the structural engineering courses following. A complete manual is generally given with each slide rule purchased, and explains in a simple manner its construction and how it is used.

Problem 1.—(*a*) A wooden roof truss has a span of 60 ft. with the distance between trusses 15 ft. Compute the weight of the truss by the two formulas given in Art. 8. (*b*) Make similar computations for a span of 40 ft. and a spacing of 12 ft.

Problem 2.—If roof trusses are equally spaced 16 ft. center to center with a span of 65 ft., and a rise of 15 ft., what weight of roof is supported by each roof truss, assuming the roof to weigh 12 lb. per sq. ft. of roof surface?

Problem 3.—(*a*) Assume the steel truss shown in Fig. 4 to have a span of 70 ft., height 18 ft., and distance between trusses 16 ft. What is the (approximate) dead load on panel point *A* considering the weight of rafters, purlins, sheathing, and shingling, same as in illustrative problem? (*b*) Make similar computations for a span of 50 ft., height 13 ft., and distance between trusses 18 ft.

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9. Dead Load on Bridges.—Before explaining the method of procedure in the determination of the dead load of highway and railroad bridges, it will be well for the student to understand

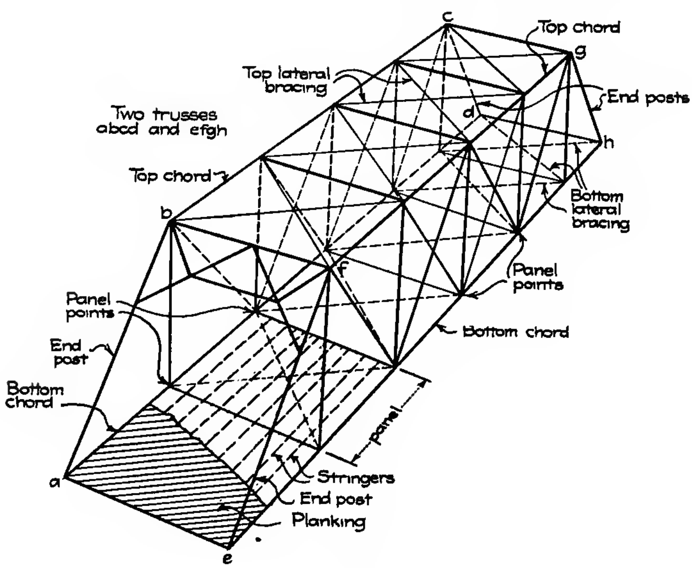

FIG. 5.

the general arrangement of the different parts of these bridges.

The floor of a highway bridge consists generally of floor beams (floor *girders*, strictly speaking), joists called stringers, and the flooring, which may be either planking or something more per-

manent. (Fig. 5). The flooring, of whatever it consists, rests upon the stringers which run parallel to the trusses (or main girders as the case may be) and these stringers are connected to and supported by the floor beams. The floor beams run at right angles to the trusses and are connected to them at the panel points. The panel points in bridge trusses are located the same as in roof trusses; that is, at the joints.

A railroad bridge differs from a highway bridge only in the construction of the floor. In the open type floor system, the ties and rails take the place of the flooring in a highway bridge,

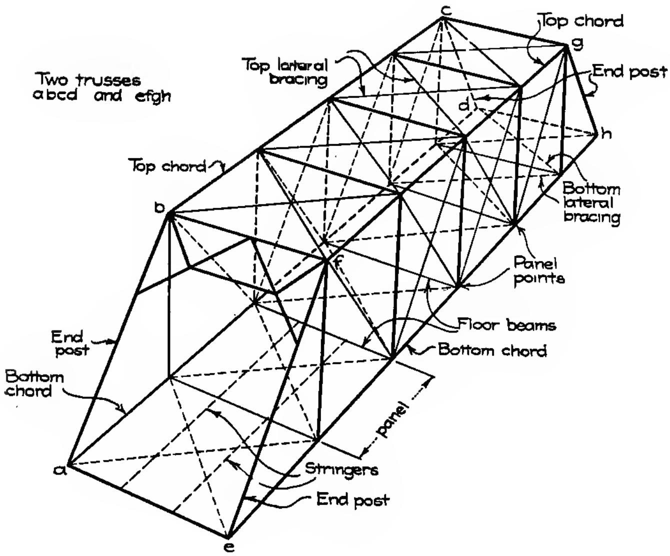

FIG. 6.

the stringers being placed below the ties and almost directly beneath the rails. The stringers are supported by the floor beams and the floor beams connected with the trusses or girders in the same manner as for highway bridges. (Figs. 6, 7, and 8.) Fig. 8 shows a *through* plate girder bridge. The girders, however, are sometimes placed closely together and the ties are notched to rest directly on the top flanges, two girders being needed foı each track, there being one at or near each rail. The stringers and floor beams are then omitted. Such a type of bridge is called a *deck* plate girder bridge.

The dead load of a highway or railroad bridge consists of the weight of floor and the weight of the trusses or girders, together with the lateral and transverse bracing which connects them.

The approximate weight of highway bridges may be determined by the use of an empirical formula in the same manner as for roof trusses. It must be borne in mind, however, that the formulas that are employed for determining the weight of high-

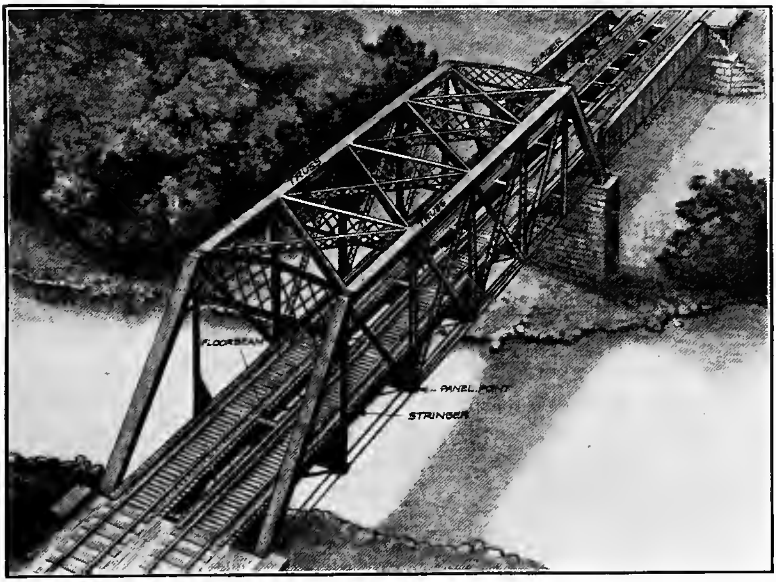

FIG. 7. FIG 8.

way bridges are likely to give values even more approximate than those used in determining the weight of roof trusses.

This may be seen if one considers how the weight of the various parts which make up the total weight actually vary in different bridges. For instance, the weight of each truss depends upon the kind of flooring used, the width and span of the bridge, the working stresses employed, the type of truss used, the horizontal distance from panel point to panel point (called the panel length) and last of all the loading for which the structure is to be designed. All engineers do not specify the same loading for the same style of bridge similarly located. They vary in their ideas concerning proper loading as they do in other things.

Moreover, the loading specified to be used in the design of a country highway bridge is not the same as that specified for a city bridge. A city bridge, for example, is liable to become densely crowded with people or vehicles, but this is a very improbable occurrence on a country highway bridge. It seems that the above discussion should convince the student how difficult it is to provide a formula for the weight of a highway bridge which will give closely approximate results.

The total weight of a highway bridge with two trusses may be roughly expressed by the following empirical formula by Merriman and Jacoby:

$$w = 140 + 12b + 0.2bL - 0.4L$$

where w = weight of bridge in pounds per linear foot of span.
b = width of bridge in feet (including sidewalks, if any).
L = span of bridge in feet.

It is customary for the bridge engineer of a railroad company to specify for the loading to be used in designing, two typical consolidation locomotives with given axle loads followed by a uniform train load. In the following formula proposed by Waddell and Hedrick especially for single-track pin-connected through Pratt truss railroad bridges, from 180 to 350 ft. span, the weight of the locomotives so specified is seen to have an important effect on the dead weight of bridge.

$$w = 8.63(L + 1.3W - 140)$$

where w = weight of all steel in the bridge in pounds per linear foot of span (not including track).
W = weight in short tons of one of the two locomotives that precede the uniform train load used in designing.
L = span of bridge in feet.

Remember the result obtained from the above formula is pounds per foot of length. To the weight thus obtained it is necessary to add from 400 to 450 lb. per linear foot for the weight of the track (including rails, guard timbers, ties, and all fastenings) in order to find the total weight per foot.

The corresponding weight of a double-track bridge is about 85 per cent more; track to be considered separately.

What has been said of highway bridges in regard to the variation in the parts that go to make up the total weight does not

exactly apply to railroad bridges since here there is less variation in the loads and in the method of design. However, unless a formula is employed which applies directly to the type of bridge to be designed, only very approximate results can be expected.

There are many more formulas in existence derived from different records of practice, but the above are as good as any for general use. Some of the formulas are to be used for bridges of a certain type, of a certain length, and other controlling conditions. If a formula can be found which has been derived from the records of bridges, similar to the one to be designed, the formula may give quite accurate results.

Since the flooring of either a highway or railroad bridge is fastened to the stringers and the stringers to the floor beams and the floor beams to the girders or trusses, the entire weight of the floor acts upon the girders or trusses at the points where the floor beams are connected to them. In through plate girder bridges these points are usually equally spaced along the girders and are called *panel points* the same as in trusses (Fig. 8). In trusses these points are at the joints which are, as a rule, equally spaced along the trusses and the weight produces only direct stress in the members. If there are two trusses in a bridge and the bridge is symmetrical about a center line, each truss carries one-half the weight of the floor.

Considering the usual symmetrical construction with two trusses, the dead load acting at a panel point on a girder or truss includes the proportional part of the weight of the girder or truss (with bracing) said to be created at the panel point in question, plus one-half the weight of the floor and track in a panel length of the bridge. This panel load may be obtained by multiplying the total weight of bridge per linear foot by the panel length and dividing by 2.

In the design of the floors of bridges, the weight per square foot of flooring is first determined, then the size of stringers may be found from the given live loading and the method of successive approximation for the dead load. With the weight of flooring and stringers, the floor beams may be designed by the same method and the total weight acting at the panel points may be computed.

Problem 4.—A highway bridge with two trusses has a span of 150 ft. and a total width of roadway of 18 ft. There is a sidewalk 5 ft. wide at each side

of the bridge outside the trusses. Determine the approximate weight of bridge in lb. per lin. ft.

Problem 5.—Span of a double-track steel truss railroad bridge is 110 ft. and the weight of each of the two locomotives specified is 142 short tons. The track weighs 400 lb. per ft. per track. Calculate the approximate weight of bridge in lb. per ft. including track.

Problem 6.—A single-track steel truss railroad bridge has a span of 175 ft., with 7 panels. Weight of each of the two locomotives specified is 142 short tons. Track weighs 450 lb. per ft. Obtain the approximate dead panel load on each truss.

Problem 7.—A plate girder single-track railroad bridge has a span of 40 ft., with 4 panels. There are two stringers in each panel, one under each rail. The girders are 14 ft. apart. Weights are as follows in lb. per lin. ft.:—track, 400; one stringer, 75; one floor beam, 100; one girder, 200. Determine the dead panel load on one girder.

ASSIGNMENT 2

CHAPTER II—*Continued*

[The general method of determining the dead load on most structures has been exemplified by its application to roofs and bridges. The dead load on many other structures may be ound in the same manner but the previous treatment is sufficient for our present needs. The computations and method in detail for any given structure will be found in the course in which such structure is specially treated. The determination of live loads will now be considered.]

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10. Snow Loads.—The snow load on structures varies with the latitude and the humidity. As a maximum it is approximately 30 lb. per horizontal square foot in Canada and northern Wisconsin, 20 lb. in the city of Chicago, 10 lb. in Cincinnati, and rapidly diminishes southward.

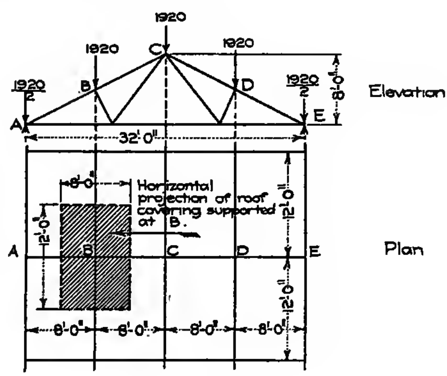

FIG. 9.

11. Snow Load on Roof Trusses.—The amount of snow which falls on an inclined roof surface cannot be greater than the amount which would fall on its horizontal projection. For this reason the maximum weight of snow which may act at a panel point of a roof truss is equal to the horizontal projection

of the inclined area of roof covering supported at a panel point, multiplied by the maximum weight of snow per horizontal square foot for the latitude in which the structure is built. For example, take the roof truss shown in Fig. 9. Span of the truss is 32 ft. and distance between adjacent trusses 12 ft. Suppose this roof truss is to be erected in the city of Chicago. The maximum snow load on panel point *B* will then be 12x8x20 =1920 lb.

On roof trusses having an angle of inclination of over 60 degrees, the snow load is often neglected if there are no snow guards, as it may naturally be expected that the snow would slide off.

It is generally assumed that a roof will not be subjected to the maximum wind load and maximum snow load at the same time. If the maximum wind load is used, only one-half the maximum snow load is taken.

Problem 8.—A roof truss erected in Northern Wisconsin, has a span of 30 ft. 3 in. and the distance between trusses is 13 ft. 9 in. The roof truss is similar to the one shown in Fig. 9. Find maximum snow load on panel point *A*.

---

12. Snow Load on Bridges.—The maximum snow load on highway bridges is never likely to occur at the same time as a full live load. For instance, a crowd of people or a great number of heavy wagons are not likely to come upon a highway bridge when the bridge supports a maximum snow load. On this account, in specifying live loads for highway bridges, the snow load per horizontal square foot is given a smaller value than it is in the case of roofs. In some standard specifications for highway bridges the snow load is entirely omitted.

Railroad bridges usually have open floors that retain but little snow and the snow load in such cases is not considered.

13. Wind Pressure.—Very few experiments have been made under the conditions of practice to determine the greatest intensity of wind upon exposed surfaces, as those of roofs and bridges. In the few experiments made, the pressure of the wind upon a plane surface normal to its direction has been found to be closely proportional to the square of the wind velocity. Professor C. F. Marvin found from his experiments on Mt. Washing-

ton in 1890 that the wind velocity and the pressure corresponding vary according to the formula

$$p = 0.004V^2$$

where p = pressure in pounds per square foot, and V = velocity of wind in miles per hour. Recent experiments made at the Eiffel Tower and at the National Physical Laboratory of England give results varying closely with the formula

$$p = 0.0032V^2$$

for square surfaces from 10 to 100 sq. ft. in area. For larger areas the pressures were somewhat less.

In estimating the pressure of wind upon large areas the variable character of the wind should be taken into account. A small part of any structure may be exposed at any time to the maximum velocity and pressure, but recent study has shown that the maximum pressure does not extend over very large areas at the same instant.

14. Wind Pressure on Roofs.—In estimating wind pressure on roofs and other inclined surfaces, the direction of the wind is assumed horizontal and the normal pressure is calculated by means of empirical formulas derived from experiments.

The following values from the experiments by Hutton give the normal pressure per square foot for different inclinations of roof surface when the wind pressure on a vertical surface per square foot is 30 lb., this being the figure which experiments show should be used in roof and bridge computations.

Inclin.	Nor. press.	Inclin.	Nor. press.	Inclin.	Nor. press.
5°	3.9	25°	16.9	45°	27.0
10°	7.2	30°	19.9	50°	28.6
15°	10.5	35°	22.6	55°	29.7
20°	13.7	40°	25.0	60°	30.0

For all inclinations greater than 60°, the normal pressure per square foot is practically 30 lb. For intermediate inclinations of the roof surface, interpolations may be made in the table.

Some engineers, however, consider 40 lb. per square foot a suitable figure to use in practice. For this value of the horizontal wind pressure, the values given in the above table for normal pressures on inclined surfaces should be multiplied by 4/3.

The empirical formula deduced by Hutton from his experi-

ments and by which the above pressures on inclined surfaces were calculated is as follows:

$$P = P_1 (\sin A)^{1.84 \cos A - 1}$$

where P = normal pressure of wind in pounds per square foot of inclined surface.

P_1 = pressure of wind in pounds per square foot on a vertical surface.

A = angle of inclination of the roof.

Wind pressure acts only upon one side of a roof at a time and the total wind load is carried by panel points on one-half of the roof truss. For example, in Fig. 9, the entire wind load is carried by panel points A, B, and C, when the wind blows from

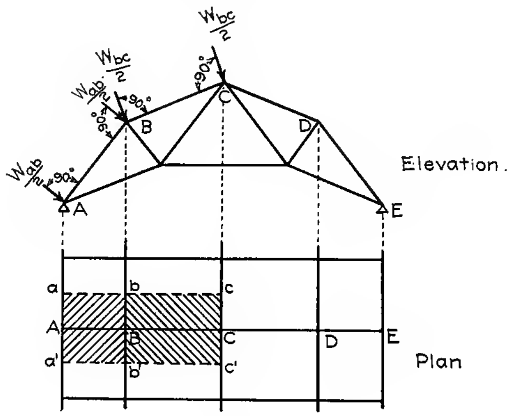

FIG. 10.

the left, and by panel points C, D, and E, when the wind blows from the right. When the wind blows from the left, panel point B, receives one-half the total load and points A and C each one-fourth.

In some roof trusses the slope of the upper chord on each side of the apex is not uniform, such as AC in Fig. 10. The panels in this truss are equal, but the inclination of panel length AB is different from that of BC, and hence, two normal unit wind pressures should be considered on each side of the roof. Suppose W_{ab} is the total pressure of wind on the area of roof surface shown in plan as $aa'\,b'b$ and W_{bc} the total pressure on the area $b\,b'c'c$. Force $\frac{W_{ab}}{2}$ will then act at panel points A and B per-

pendicular to the member AB and $\frac{W_{bc}}{2}$ will act at panel points B and C perpendicular to the member BC, as shown.

Pressures on other than plane areas are variously estimated about as follows:—on cylindrical surfaces, 60 to $66\frac{2}{3}$ per cent of that on plane areas; on octagonal prisms, 70 per cent; on concave shaped areas 125 to 150 per cent.

Illustrative Problem.—What is the normal wind pressure per sq. ft. on a roof making an angle of 37° with the horizontal when the wind pressure on a vertical surface is 30 lb. per sq. ft.? Get result by table and also by formula.

$$\begin{array}{r} 25.0 \\ \underline{22.6} \\ 2.4 \end{array}$$

$$(2/5)(2.4) = \begin{array}{r} 1.0 \\ \underline{22.6} \\ 23.6 \end{array} \text{ lb. per sq. ft. (by table) } \textit{Answer.}$$

$$\begin{aligned} P &= P_1 (\sin A)^{1.84 \cos A - 1} \\ &= (30)(\sin 37°)^{1.84 \cos 37° - 1} \\ &= (30)(\sin 37°)^{0.47} \\ &= 23.6 \text{ lb. per sq. ft. (by formula) } \textit{Answer.} \end{aligned}$$

$$\begin{array}{rl} \text{nat. cos. } 37° & = 0.7986 \\ (1.84)\ (0.7986) & = 1.47 \\ \text{log. sin. } 37° & = 9.7795 - 10 \\ (9.7795 - 10)\ (0.47) & = 4.5964 - 4.7 \\ & = 9.8964 - 10 \\ \text{log. } 30 & = \underline{1.4771} \\ \text{log. of answer} & = 1.3735 \\ \text{number corresponding} & = 23.6 \end{array}$$

Several formulas are in existence for determining wind pressure on inclined surfaces. The best known of these are Duchemin's, Hutton's and the Straight Line formulas. Hutton's formula has perhaps been more used in practice than any other, but of late a review of Hutton's apparatus and experiments (Hutton's mathematical papers) has been made, and serious doubts have arisen upon the accuracy of this formula. Duchemin's formula is based upon carefully conducted experiments, gives larger values, and is now considered by many engineers more reliable than Hutton's. The Straight Line formula is preferred by many on account of its simplicity, and gives results which agree quite closely with experiments.

Duchemin's formula is

$$P = P_1 \frac{2 \sin A}{1 + \sin^2 A}$$

where P, P_1, and A are the same as in Hutton's formula.

The Straight Line formula is

$$P = \frac{P_1}{45} A$$

where P, P_1, and A are the same as in Hutton's formula.

Problem 9.—What is the normal wind pressure per square foot on a roof making an angle of 42° with the horizontal, when the wind pressure on a vertical surface is 40 lb. per sq. ft.? Get result by table and also by Hutton's formula.

Problem 10.—What is the wind load on each panel point on the truss shown in Fig. 9 if the wind is blowing from the left. Wind pressure is 30 lb. per sq. ft. on a vertical surface. Use table derived from Hutton's formula.

Problem 11.—Solve problem 9 using Duchemin's formula.

Problem 12.—Solve problem 9 using the Straight Line formula.

---

15. Wind Pressure on Bridges.—The wind pressure on bridges is assumed to act in either direction horizontally.

The wind pressure on highway bridge trusses (or girders) is usually taken at 30 lb. per square foot on the exposed surface of all trusses and floor as seen in elevation, in addition to a horizontal live wind load of 150 lb. per foot moving across the bridge. Also, a second set of computations are made, using generally 50 lb. per square foot on the exposed surface of all trusses and the floor system, with *no* moving wind load. The greater result is used in proportioning the truss members.

For railroad bridges the static wind load is usually the same as for the two cases considered for highway bridges, but instead of a moving wind load of 150 lb. per linear foot of span mentioned in the first case, a moving wind pressure of 30 lb. per square foot is generally considered to act on a train moving across the bridge having 10 ft. average height, beginning 2 ft. 6 in. above the base of rail.

Wind pressure on bridges is carried by lateral trusses placed between and connecting the vertical trusses. (Fig. 5). The

stresses resulting from wind pressure will be treated in "Bridge Trusses, Part 1," Course 412.

16. Loadings Specified for Highway Bridges.—The live load to be used in the design of a given bridge is generally specified by the chief engineer, who is also responsible for the design of the bridge and its construction. The members of a highway bridge truss generally receive their greatest stress when a densely packed crowd of people covers the roadway and sidewalks and when the car tracks, if any, are covered by electric cars. The members constituting the floor system under the roadway are most highly stressed when heavy axle loads pass over the bridge; for instance, such loads as road rollers and traction engines.

Standard specifications for the design of highway bridges have been drawn up by a number of prominent bridge companies and by the bridge departments of many railroad companies. A few consulting engineers have also published specifications of this character. These specifications take up, among other things, the amount and kind of live load to be used in the designing of the different types of highway bridges. An allowance is made in each case for the probable increase in axle loads for a number of years in the future. Consequently, the loads specified are not the actual loads, and the spacing is such as to give ease in computation; the object of specifications being to make sure the members that are put into the bridge will be strong enough to carry any loads that are likely to come upon the bridge during its lifetime.

The following is taken from the "Specifications for Bridges carrying Electric Railways," adopted by the Massachusetts Railroad Commission:

"Stringer spans and the floor system of all trusses or girders shall be proportioned to carry a double-truck car weighing when loaded 50 tons with a total wheel-base of 25 ft. and a wheel-base for each truck of 5 ft.

"Trusses and girders shall be proportioned to carry one car of the above type, or a uniformly distributed load, on each track. This uniform load shall be varied according to the length which has to be loaded by it to produce the maximum stress in the member in question. If this "loaded length" is 100 ft. or less, the load shall be 1500 lb. per linear foot of track; and if the "loaded length" is 300 ft. or over, the load shall be 1000 lb. per linear foot of track, and proportionately for intermediate lengths.

"In highway bridges carrying electric roads the above specifications shall apply with reference to the loads upon the railway track. In addition, the following moving loads shall be assumed upon the highway floor:

"(a) For city bridges, subject to heavy loads:

"For the floor and its supports, a uniform load of 100 lb. per square foot of surface of the roadway and sidewalks, or a concentrated load of 20 tons on two axles 12 ft. apart, with 6 ft. between wheels. In computing the floor beams and supports, the railway load shall be assumed, together with either (1) this uniform load extending up to within 2 ft. of the rails, or (2) the above-described concentrated load alone.

"For the trusses or girders, 100 lb. per square foot of floor surface for spans of 100 ft. or less, 80 lb. for spans of 200 ft. or over, and proportionately for intermediate spans. This uniform load is to be taken as covering the floor up to within 2 ft. of the rails.

"(b) For suburban or town bridges, or heavy country highway bridges:

"For the floor and its supports, a uniform load of 100 lb. per square foot, or a concentrated load of 12 tons on two axles 8 ft. apart; these loads to be used as described under (a).

"For the trusses or girders, 80 lb. per square foot of floor surface for spans of 100 ft. or less, and 60 lb. for spans of 200 ft. or more, and proportionally for intermediate spans; to be used as described under (a). See (d).

"(c) For light country highway bridges:

"For the floor and its supports, a uniform load of 80 lb. per square foot; this load to be used as described under (a). See (d).

"For the trusses or girders, 80 lb. per square foot of floor surface for spans of 75 ft. or less, and 50 lb. for spans of 200 ft. or more, and proportionally for intermediate spans; to be used as described under (a).

"(d) All parts of the floor of a highway bridge should also be proportioned to carry a road roller weighing 15 tons, and having three wheels or rollers, the weight on the front roller being 6 tons, and the weight on each rear roller to be 4.5 tons. The width of the front roller is to be taken as 4 ft., and of each rear roller 20 in.; the distance apart of the two rear rollers to be 5 ft. center to center, and the distance between front and rear rollers 11 ft. center to center."

Problem 13.—A city highway bridge with paved roadway and sidewalks and 2 electric car tracks has a span of 140 ft. 6 in. Its roadway is 40 ft. in the clear and each of its two sidewalks has a clear width of 6 ft. The loaded length to produce maximum stress in a given member of one of the trusses is 80 ft. Assume the load on each track to cover a width of 9 ft. Find the total live load per lin. ft. according to the specifications adopted by the Massachusetts Railroad Commission.

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17. Conventional Load Systems for Railroad Bridges.—Axle Loads.—It is customary, as before mentioned, for the bridge engineer of a railroad company to specify for use in designing, two typical consolidation locomotives, with given axle loads and spacing, followed by a uniform train load. The bridge engineer of each railroad company formerly selected a loading based on the actual weights of the heaviest locomotives in service on his particular road. Such a great variety of specifications in existence rendered the calculation of stresses very troublesome. Considerable progress has been made toward the adoption of a simple *standard* system which will give approximately the same results and whereby the calculations may be simplified. Cooper's standard train loading, Class E 50, is shown in Fig. 11.

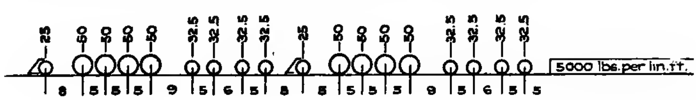

FIG. 11.

Other loadings are known as class E 30, class E 40, and class E 60, etc. The spacing is the same for all classes, and the loads of any two classes have the same ratio as their class numbers. For example, the loads of class E 30 are three-fifths of those of class E 50, etc. Any stresses due to these loadings are also proportional to their class numbers.

Equivalent Uniform Load System.—An equivalent uniform load for calculating the stress in a given truss is one so chosen as to cause practically the same stresses as those due to the specified axle loads. This method is used to a considerable extent. Its purpose is to simplify the computation of the stresses. For ordinary bridges, however, there is really not much gained in this respect and the selection of such equivalent loads is somewhat troublesome. For short girder spans its accuracy is hardly

sufficient. For spans over 100 ft. in length it will give results accurate to within 2 or 3 per cent for most members, if the load is properly selected. The method of determining the proper equivalent load will be taken up in "Bridge Trusses, Part 1," Course 412.

Excess Panel Loads.—A few railroad bridge departments specify a uniformly distributed trainload plus a concentrated load so placed as to produce the greatest effect in each case. The excess load may be regarded as representing approximately the difference in weight between a locomotive and that of a uniform load for the same length. It is sometimes assumed as equal to the uniform load over a distance of 10 ft.

The use of two concentrated excess loads is sometimes specified. These loads are usually required to be placed 50 ft. apart and may occupy any position in the uniform trainload.

ASSIGNMENT 3

CHAPTER III

PRINCIPLES OF STATICS

[The student by this time should understand the distinction between the live and dead loads on any structure and should have a knowledge of the method of procedure in determining the live and dead loads on such structures as roofs and bridges. The next step in design is to compute the reactions for these structures, but to do this the principles of statics should be understood.]

The term *statics* is applied to that branch of mechanics which deals with the equilibrium of forces; that is, where forces are prevented from causing motion by being resisted by equal and opposite forces. *Kinetics* is the term applied to that branch of mechanics which deals with forces *causing* motion.

Problems in statics may be solved either algebraically or graphically. In the algebraic method, forces are represented by symbols, and equations are employed to solve for unknowns. In the graphical method forces are represented by lines, and problems are solved by the aid of the drawing board and ordinary instruments. Both methods will be explained in this course, since in some problems it is easier to employ the algebraic method while in others the graphical method is more simple.

18. Elements of a Force.—A force acting upon a body is completely specified when its *direction*, *point of application*, and *magnitude* are known. A straight line may be made to represent these elements by making its direction (meaning position of line of action and direction on this line of action) the same as that of the motion which the force imparts, or tends to impart to the body; one end of it, the point of application; and its length proportional to the magnitude of the force.

Thus, the position of the line and the arrowhead in Fig. 12 indicate completely the direction of the force exerted upon the

body B; the point where the straight line meets the body is its point of application; and the length of the line its magnitude.

Forces are given in pounds while the lengths of lines are measured in inches. If scale of force be 5000 lb. to an inch, a line 0.20 in. long will represent a force of 1000 lb.; that is, 5000x0.20 =1000. A line 1.55 in. long will represent a force of 7750 lb. The method of finding the length of line to represent a given force is to divide the given force in pounds by the number of pounds in 1 in. of length. Thus, $\frac{7750}{5000}=1.55$ in. An engineer's scale which is divided into 20ths, 30ths, 40ths, 50ths, 60ths, and 80ths of an inch should be used in all the graphical work which

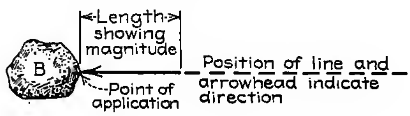

FIG. 12.

follows. It will also be needed in all the other courses in structural designing. The part of the scale divided into 50ths should be used for the present in laying off the lengths of lines to represent forces. Each division equals 1/50=2/100 of an inch. 1.55 in. on this scale would thus equal 50 divisions (1 in.) plus 27 1/2 divisions (55/100 of an inch) = 77 1/2 divisions. The same result can be obtained much more easily in the following manner: $\frac{1.55}{.02}=77\ 1/2$. Notice that the small numbers along the scale are an aid in scaling.

Problem 14.—If the scale is 1 in. = 100 lb., how long are the lines to represent forces of 952, 1455, and 45 lb.? How many divisions on the part of the scale graduated to 50ths?

Problem 15.—If the scale is 1 in. = 1500 lb., how large are the forces represented by 2.15 in. and 0.95 in.?

---

19. Concurrent and Non-concurrent Forces.—Forces are said to be *concurrent* when their lines of action meet in a point; *non-concurrent* when their lines of action do not so meet.

Forces may be in the same plane or in different planes; that is,

coplanar, or *non-coplanar* forces. In such structures as roof and bridge trusses we shall need to deal only with forces whose lines of action lie in the same plane.

20. Equilibrant of Forces.—When a number of forces act upon a body and the body does not move, or if moving does not change its state of motion, then the forces considered are said to be in *equilibrium*. Any one of the forces balances all the other forces and it is called the *equilibrant* of those other forces.

21. Resultant of Forces.—A single force which would produce the same effect as a number of forces is called the *resultant* of those forces. The process of finding the single force is called *composition*.

It is evident from the above that the equilibrant and resultant of a number of forces are equal in magnitude, act along the same line, but are opposite in direction.

22. Components of a Force.—Any number of forces whose combined effect is the same as that of a single force are called *components* of that force. The process of finding the components is called *resolution*.

23. Moment of a Force.—The moment of a force about a point is the tendency of the force to produce rotation about that point. It is equal to the magnitude of the force multiplied by the perpendicular distance of its line of action from the given point. The point about which the moment is taken is called the *origin* (or *center*) *of moments*, and the perpendicular distance from the origin to the line of action is called the *lever arm* (or *arm*) of the force. When a force tends to cause rotation in the direction of the hands of a clock it is considered *positive*, and in the opposite direction, *negative*.

24. Couple.—A couple consists of two equal and parallel forces, opposite in direction, and having different lines of action. The *arm* of the couple is the perpendicular distance between the lines of action of the two forces. The *moment of a couple* about any point in the plane of the couple is equal to the algebraic sum of the moments of the two forces, composing the couple, about that point. (Algebraic sum of the moments means the sum of the moments of the forces, considering positive moments *plus* and negative moments *minus*).

In Fig. 28 consider P_1 equal and parallel to P and let P and P_1 be the only forces acting upon the body. P and P_1 will cause rotation of the body and the amount of this rotation about any

point in the same plane as the couple (provided the body is pivoted at that point) is measured by the algebraic sum of the moments of P and P_1 about that point. Consider the body to be pivoted at o in the same plane with the forces. The moment of P about the point o is $P(r+r')$ and the moment of P_1 about the same point is P_1r', $(r+r')$ and r' being the lever arms of P and P_1 respectively. The moment of P_1 is positive and the moment of P is negative. Then the moment of the couple is equal to $P_1r' - P(r+r') = -Pr$ (since P is equal to P_1). The *moment of a couple* is thus equal to one of the forces multiplied by the perpendicular distance between the lines of action of the forces. Since o is any point in the plane of the couple, it is evident that the moment of the couple is independent of the origin of moments. This means that the rotation would be the same no matter where the body should happen to be pivoted in the plane of the forces.

25. The Force Triangle.—In Fig. 13 let forces F_1 and F_2, which are concurrent forces acting at the point o, be represented in magnitude and direction by oa and ob respectively. From b draw bc parallel to oa and from a draw ac parallel to ob. Join the

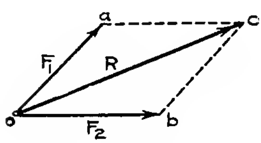

FIG. 13.

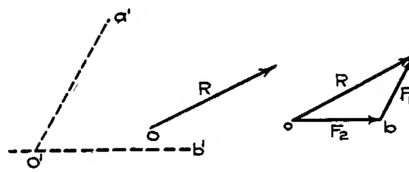

FIG. 14.

point of intersection c with o. The line oc is the magnitude of a single force R which will produce the same effect as the forces F_1 and F_2. Thus, R is the resultant of F_1 and F_2. A force equal and opposite in direction to R would be the equilibrant of F_1 and F_2, since it would hold them in equilibrium. F_1 and F_2 are components of R.

It is not necessary to construct the entire parallelogram since the triangles on the opposite sides of the diagonal are equal; $bc = oa$; $ac = ob$. Either of these triangles is called a force triangle and either one, if constructed, is sufficient to give the value of the resultant and the equilibrant of forces F_1 and F_2.

If only R is given at the point o, Fig. 14, and it is desired to obtain two components of R parallel to the lines $o'a'$ and $o'b'$,

then oc is first drawn equal in magnitude and parallel to R, ob is drawn from o parallel to $o'b'$, and cb is drawn from c parallel to $o'a'$ and the lengths of the lines ob and bc, when scaled from the drawing, give the magnitude of the two components desired.

It should be noted that if R were opposite in direction, the direction of the forces would follow in order around the sides of the triangle. A force opposite to R would be the equilibrant of the forces F_1 and F_2 and the three forces would be in equilibrium. This leads us to the following propositions:

1. If three forces be represented, in magnitude and direction, by the three sides of a triangle taken in order, then, if these forces be simultaneously applied at one point, they will balance each other.

2. Conversely, three forces which, when simultaneously applied at one point, balance each other, can be correctly represented in magnitude and direction by the three sides of a triangle taken in order.

The construction of the force triangle is based upon the following accepted truths:

1. The velocity (or rate of motion) imparted to a given body can be measured by the distance the body is moved in one second.

2. If a body has two or more velocities imparted to it simultaneously, it will move so as to preserve them all.

3. Forces are proportional to the velocities which they will impart to a given body in a unit of time.

In Fig. 15 suppose a body situated at O to have two motions imparted to it simultaneously, one of which would carry it to A in one second, and the other to C in one second. Imagine the body to move in obedience to the first alone during one second. It would arrive at A at the end of the second. Now imagine the body to move in obedience to the second motion instead of the first. The body would arrive at B at the end of the next second, where BC is equal and parallel to OA. If the two motions were applied simultaneously instead of successively, the body would preserve them both and, consequently, would travel along the diagonal OB. OA and OC may represent any motions imparted to the given body, hence, the resultant motion may always be represented as one side of a triangle with the separate motions imparted as the other two sides. As stated above, forces are proportional to the velocities which they impart to a given body in a unit of time. Consequently, lines OA and OC represent the

forces (although to different scale) which would produce the motions represented by OA and OC and, since the resultant velocity is shown by OB, OB also represents the resultant force of the two forces OA and OC.

The student can prove the law of the force triangle experimentally by means of two spring balances and a known weight. On a smooth vertical surface suspend the two balances at about the same height and a few feet apart. Fasten the weight to a small ring by means of a short chord. In a similar manner connect the ring with the hooks on the balances. The forces will be in equilibrium and their lines of action will all intersect at the center of the ring; that is, the system of forces will be

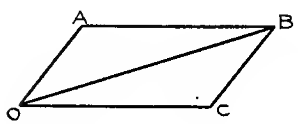

FIG. 15.

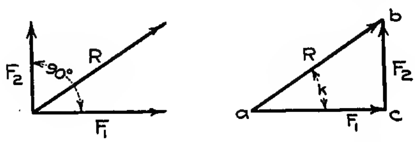

FIG. 16.

concurrent. The resultant of the pull in the balances determined graphically on the vertical surface should be equal and opposite in direction to the weight of the suspended body.

If the angle between the lines of action of two forces is 90° then the algebraic method should be used. In Fig. 16 the angle between the lines of action of F_1 and F_2 is 90°. It is required to find the value of the resultant R. Since abc is a right triangle

$$\overline{ab}^2 = \overline{ac}^2 + \overline{bc}^2$$

or $$R = \sqrt{F_1^2 + F_2^2}$$

The direction of the resultant R is decided by the angle K. K may be determined as follows:

$$\tan K = \frac{bc}{ac} = \frac{F_2}{F_1}$$

If R is known and the components F_1 and F_2 are required:

$$F_1 = R \cos K$$
$$F_2 = R \sin K$$

Problem 16.—The magnitude of F_1 in Fig. 13 is 600 lb., and that of F_2 is 500 lb. The direction of F_1 makes an angle of 60° with the direction of F_2. Find the magnitude of their resultant. Use the scale 1 in. = 200 lb.

Problem 17.—The magnitude of R in Fig.14 is 800 lb. Lines $o'a'$ and $o'b'$ make an angle of 60° with each other. Determine the magnitude of two components of R parallel to $o'a'$ and $o'b'$. R makes an angle of 30° with the line $o'b'$. Use the scale 1 in. =250 lb.

Problem 18.—Suppose F_2 in Fig. 16 equals 550 lb. and F_1 equals 1420 lb. Find the magnitude of the resultant R. F_2 and F_1 act at right angles to each other. Solve algebraically.

Problem 19.—Suppose the wind load on panel point B in Fig. 9 is 5000 lb. acting normal to the surface. Find the horizontal and vertical components of this load. Solve algebraically.

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26. The Force Polygon.—The construction of the force polygon is based upon the same three accepted truths that were presented under the discussion of the force triangle. The same principles apply whatever be the number of concurrent forces imparted to a body simultaneously. The following propositions are deduced in the same way as for the triangle of forces.

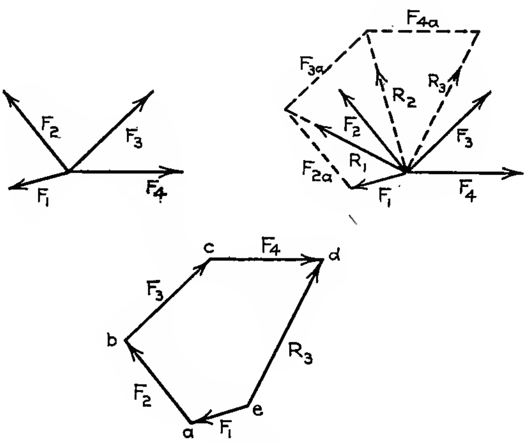

FIG. 17.

1. If any number of forces be represented in magnitude and direction by the sides of a polygon taken in order, then, if these forces be simultaneously applied at one point, they will balance each other.

2. Conversely, any number of forces which, when simultaneously applied at one point, balance each other, can be correctly

represented in magnitude and direction by the sides of a polygon taken in order.

The above proposition may be shown to be true, if we consider the construction of the force polygon in Fig. 17. The resultant of the four concurrent forces F_1, F_2, F_3, and F_4 is to be found. This will be done by finding the resultant of two forces, then by combining this resultant with a third force to find a second resultant, and so on until all the forces are combined and the resultant of all the forces determined.

The resultant of the forces F_1 and F_2 is R_1 determined by the force triangle $R_1F_1F_{2a}$; F_{2a} being drawn parallel to F_2. In the same manner R_2 is the resultant of R_1 and F_3, also R_3 is the resultant of R_2 and F_4. R_3 is then the resultant of the four forces F_1, F_2, F_3, and F_4. F_1, F_{2a}, F_{3a}, F_{4a}, and R_3 form a closed polygon. F_{2a}, F_{3a}, and F_{4a} are parallel and equal in magnitude to forces F_2, F_3, and F_4 respectively, being drawn so. A closed polygon can, therefore, be drawn by drawing in succession, lines parallel and equal to the given forces, each line beginning where the preceding one ends and extending in the same direction as the force it represents. The line joining the initial to the final point represents the resultant in magnitude and direction. The diagram *abcde* shows the polygon as it is generally drawn with the diagonals omitted.

The arrow (or direction) of R_3 opposes the arrows of the other forces in following around the triangle. A force equal and opposite to R_3 would be the equilibrant of the forces or, in other words, the forces would be in equilibrium. Thus follows the propositions stated above which may be expressed in a somewhat different manner by saying that if a closed force polygon can be drawn for a system of concurrent forces, the forces considered are in equilibrium; and conversely, that for a system of concurrent forces in equilibrium the force polygon must close.

It makes no difference in what order forces are arranged in the force polygon since the magnitude and direction of the resultant obtained will be the same. The student should verify this statement by making several force polygons with the forces in different order.

The force triangle is but a special case of the force polygon; that is, when there are only three forces to consider. The term force polygon as commonly used includes the force triangle as well. We shall use it hereafter in this general sense.

Illustrative Problem.—The crane truss shown in Fig. 18 is loaded with 3000 lb. at C. Determine the stresses in the boom CD; the tie BC; the mast BD; and the stay AB.

$$\overline{BC}^2 = \overline{8}^2 + \overline{15}^2 \qquad \overline{CD}^2 = \overline{20}^2 + \overline{15}^2 \qquad \overline{AB}^2 = \overline{12}^2 + \overline{9}^2$$

$$BC = 17 \qquad CD = 25 \qquad AB = 15$$

At the point C three forces are acting; namely, the 3000 lb. load, the stress F in the tie BC, and the stress F_1 in the boom CD. Draw the force polygon bcd by laying off the vertical lind. bd equal to 3000 lb. (since weight always acts vertically) and drawing bc and dc parallel to F and F_1 respectively.

Since there is equilibrium in the crane truss the forces acting at the point C are in equilibrium. Hence, the force polygon should close and the forces should act in order around the polygon. If the drawing is made to scale,

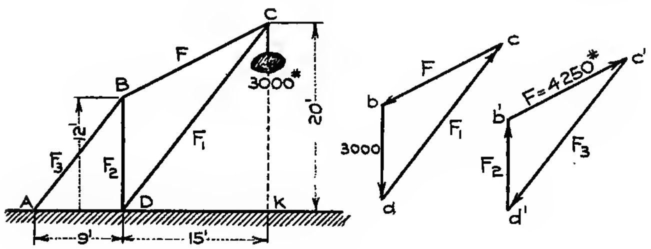

FIG. 18.

the lines bc and dc represent directly the magnitude and direction of F and F_1. It should be noticed that triangle bcd is similar to triagnle BCD and it is not necessary to construct a separate force polygon if the crane truss is drawn to some scale in the first place. For example, if the scale used for drawing the truss is 1 in. $=2$ ft. then $BD=6$ in. But BD represents a force of 3000 lb., hence, the scale used for determining the forces should be 1 in. $=$ 500 lb.

F and F_1 may also be solved algebraically as follows:

$$\frac{BC}{BD} = \frac{17}{12} = \frac{F}{3000}$$

$$F = 4250 \text{ lb.}$$

$$\frac{CD}{BD} = \frac{25}{12} = \frac{F_1}{3000}$$

$$F_1 = 6250 \text{ lb.}$$

It will be noticed that the stress F_1 acts toward the point C or, in other words, it is the stress acting against the shortening of the member CD, thus denoting compression. The force F is the stress acting against the lengthening of the member BC, thus denoting tension. We know this to be true, and we have then a general rule, that, when a force is shown by the force polygon to act toward the point of application of the forces, the stress caused is compression, and, when a force is shown to act away from the point of application of the forces, the stress caused is tension.

A force polygon $b'c'd'$ should next be drawn for the forces at the point B. The force F is now known and the two unknown forces F_2 and F_3 may be found in the same manner as the forces F and F_1 were obtained from the force 3000. In fact it should be remembered that when the forces of a concurrent system in equilibrium are all known except two, the magnitudes and directions of these two forces may be deteimined if only their lines of action are known.

Since the tangents of the two angles BAD and CDK are each equal to 4/3, the angles themselves are equal and AB is parallel to CD. Thus, the force polygon drawn for the three forces F, F_2, and F_3 is similar to triangle BCD. If the crane truss is drawn to scale no separate force polygon is needed. BD and CD, if properly scaled, will give the magnitude and direction of F_2 and F_3. However, it is not even necessary to scale the forces in this case since it is evident that F_1 and F_3 are equal in magnitude and that F_2 is equal to the weight; that is, 3000 lb.

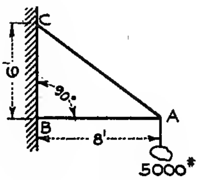

FIG. 19.

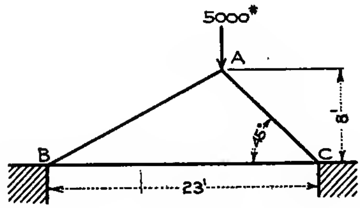

FIG. 20.

We know F to be tension, hence, we should represent it as acting away from the point b'. The arrows must follow in order around the triangle, consequently, F_2 is compression and F_3 is tension.

F_2 and F_3 may also be solved independently as follows:

$$\frac{BC}{BD}=\frac{17}{12}=\frac{4250}{F_2}$$

$F_2=3000$ lb. (same as the weight).

$$\frac{BC}{BD}=\frac{17}{25}=\frac{4250}{F_3}$$

$F_3=6250$ lb. (same as F_1).

Answers
$F\ =4250$ lb. (tension)
$F_1=6250$ lb. (compression)
$F_2=3000$ lb. (compression)
$F_3=6250$ lb. (tension)

Note.—The student should now get some practice in using parts of the engineer's scale other than the part divided into 50ths. The part of the scale divided into 20ths should be used in problems 21, 22, and 23. 1 in. for these problems will equal 2000 lb. The inch is divided into 20 parts so each division equals 100 lb. 30 lb., for instance, is 1/3 of a division and the student must get so he can judge such distances accurately by eye. The 50ths scale could be used, but it is not quite as convenient.

Problem 20.—Draw at least 4 different force polygons for the forces shown in Fig. 17. Transfer the forces (in magnitude and direction) to another sheet by means of compass, triangles, and dividers.

Problem 21.—The weight of 5000 lb. suspended at A in Fig. 19, is supported by the two pieces AB and AC, attached to the wall at B and C. Find the stresses in AB and AC both algebraically and graphically. Use scale 1 in. = 2000 lb.

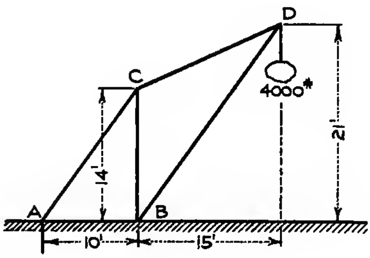

FIG. 21.

Problem 22.—Find the stresses, both algebraically and graphically, in the rafters AB and AC of the simple triangular roof truss shown in Fig. 20, the load at the apex being 5000 lb. Use scale 1 in. = 2000 lb.

Problem 23.—The crane truss shown in Fig. 21 is loaded with 4000 lb. at D. Determine the stresses, both algebraically and graphically, in the boom BD; the tie CD; the mast BC; and the stay AC. Use scale 1 in. = 2000 lb.

ASSIGNMENT 4

CHAPTER III—*Continued*

27. Equilibrium of Concurrent Forces.—Suppose a number of forces in equilibrium and acting at a single point on a given body be resolved into components in two directions at right angles to each other; horizontal and vertical, for example. The body will evidently be in equilibrium under the action of these component forces since they produce the same effect as their resultants. Moreover, the component forces along each line must balance or the body would move along that line.

We already know that the condition of equilibrium for a concurrent system is that the force polygon must close. The condition of equilibrium may now be stated in a different way, by saying that the algebraic sums of the components of the forces along each of two lines at right angles to each other must equal zero. (By the algebraic sum is meant the sum of the forces considering one direction plus and the opposite direction minus.) We call the first condition *graphical* and the second *algebraic*.

Let ΣH represent the algebraic sum of the components along a horizontal line and let ΣV represent the algebraic sum of the components along a vertical line. Then a special case of the above condition of equilibrium would be $\Sigma H=0$ and $\Sigma V=0$.

Problems in the equilibrium of concurrent forces may be solved either graphically or algebraically if the number of unknowns is not greater than two. In the graphical method the two unknowns may be determined by the closure of the force polygon, while in the algebraic method the two unknowns may be found by means of two independent equations made possible by the conditions above stated. The two unknowns which may be determined in any given case are the magnitude and direction of one force, the magnitudes or directions of two forces, or the magnitude of one and the direction of the other.

Of the four cases mentioned, only two are of sufficient importance to need special consideration; namely, where the following unknowns are required: (1) The magnitude and direction of one force, and (2) the magnitudes of two forces. The method of

solving these cases graphically should be clear from the study of Article 26. Some explanation is required, however, in regard to the manner of treating case (1) algebraically. Case (2) will be treated algebraically by an illustrative problem.

Suppose the resultant is required of a number of concurrent forces. This is Case (1) referred to above; that is, a set of concurrent forces may be considered in equilibrium, these forces being all known except one, which is unknown in both magnitude and direction. Resolve each force algebraically into components F_x and F_y parallel to lines X and Y respectively; lines X and Y being any lines at right angles to each other and called rectangular axes. Let R_o represent the resultant of all the forces acting at the given point; ΣF_x the algebraic sum of the components along the line X; and ΣF_y the algebraic sum of all the forces along the line Y. ΣF_x will then be the component of R_o along the line X and ΣF_y will be the component along the line Y. From the principles shown in the construction of the force triangle, the magnitude of R_o is given by the formula:

$$R_o = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

and its direction by

$$\tan K = \frac{\Sigma F_y}{\Sigma F_x};$$

K being the angle between the resultant R_o and the line X. Particular attention should be paid to the signs of ΣF_x and

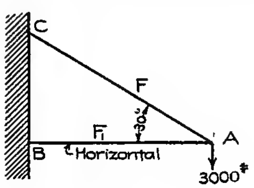

FIG. 22.

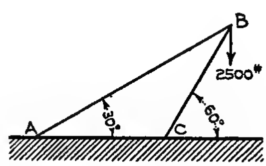

FIG. 23.

ΣF_y in order to completely determine the direction of the resultant. (Σ means *algebraic sum of.*)

Illustrative Problem.—Determine the stress algebraically in the members AC and AB, shown in Fig. 22. This is Case (2) above mentioned; that is, the concurrent forces at A are in equilibrium, these forces being all known in line of action, but two unknown in magnitude.

Since F_1 is horizontal, the vertical component of F must equal 3000 lb. in order that ΣV may equal zero at the point A.

$$F \sin 30° = 3000$$
$$F = 6000 \text{ lb. (tension)}$$

In order that $\Sigma H = 0$

$$F_1 = F \cos 30°$$
$$F_1 = 5200 \text{ lb. (compression).}$$

Problem 24.—Determine the stress algebraically in the members AB and BC, shown in Fig. 23. (The problem is solved by simultaneous equations. From $\Sigma H = 0$ and $\Sigma V = 0$ at the point B, obtain two equations each containing the two unknown forces and then solve.)

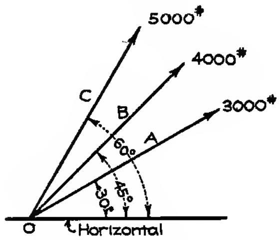

FIG. 24.

Problem 25.—Concurrent forces A, B, and C, shown in Fig. 24, make angles of 30°, 45°, and 60° respectively with the horizontal. Force $A = 3{,}000$ lb. Force $B = 4000$ lb. Force $C = 5000$ lb. Determine algebraically the magnitude of the resultant to the nearest 100 lb., and the angle it makes with the horizontal to the nearest 1/2 degree.

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28. Equilibrium of Non-concurrent Forces (*Graphical*).—When several forces lying in the same plane and acting on a given body have different points of application, so that their lines of action do not intersect in the same point, the magnitude of the resultant may be found graphically by compounding the forces in the same manner as in concurrent systems, the actual constructions, however, being not quite so simple. Two of the forces may be produced until they intersect and their resultant found, then the resultant of these two forces compounded with a third, then the resultant of the first three compounded with the fourth, and so on until the resultant of all has been found.

For example, it is required to determine the resultant of the four forces shown in Fig. 25 which act on a given body. Produce forces F_1 and F_2 until they meet at the point o. The resultant

of these forces is R_1, the magnitude and direction of which is determined by the force triangle *abc* in Fig. 26. Produce R_1 until it intersects the third force F_3 at *m*. R_2 is the resultant of F_3 and R_1, determined by the force triangle *acd*. Produce R_2 until it intersects the force F_4 at *n*. R_3 is the resultant of F_4 and R_2 determined in the same manner as before; that is, by the force triangle *ade* and, consequently, R_3 is the resultant of the four given forces.

It will be noticed in Fig. 26 that we have actually constructed a force polygon for the given forces and the resultant of all the forces is represented by the closing line *ae*. We have, then, the same general rule for non-concurrent forces as we did for concurrent forces; namely, that the magnitude of the resultant of

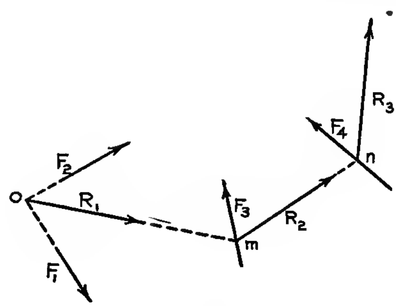

Fig. 25.

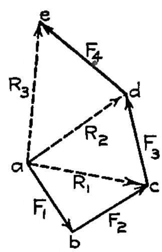

Fig. 26.

any number of forces acting in the same plane may be found by constructing the force polygon and scaling the closing side. The line *ae* also shows the direction of the resultant R_3, but notice carefully that it does not give a point on its line of action. We cannot determine a point in the line of action of the resultant unless we make the construction shown in Fig. 25 (or its equivalent) and determine a point corresponding to *n*. A force equal and opposite to R_3 and having the same line of action will balance the forces acting and the system will be in equilibrium.

It should be remembered that the above discussion refers to any number of forces acting in the same plane and applied to a rigid body at different points in such a way that their lines of action do not intersect in the same point. If their lines of action *do* intersect in the same point we have the case of concurrent forces already explained. A good example of a non-concurrent system in equilibrium is that of a roof truss subjected to

snow, wind, and dead loads. The reactions at the supports balance the above forces.

Problem 26.—Determine the magnitude, direction, and a point on the line of action of the resultant of 4 non-concurrent forces, F_1, F_2, F_3, F_4, which if taken in order make angles of 60° with each other; that is, angles between F_1 and F_2, F_2, and F_3, F_3 and F_4, are each equal to 60°. The forces considered in order have magnitudes of 850 lb., 900 lb., 720 lb., and 640 lb. Scale to be used 1 in. = 300 lb. Use part of scale divided into 30ths. Arrange the forces so that the intersection will come within the limits of the drawing.

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29. Equilibrium of Non-concurrent Forces (*Algebraic*).—In considering concurrent forces we found that the algebraic conditions of equilibrium can be stated by the expressions $\Sigma H = 0$ and $\Sigma V = 0$, if we use these expressions in a general sense. These conditions also apply to non-concurrent forces for the same reason as before, that the algebraic sum of the component forces along each of two lines at right angles to each other must

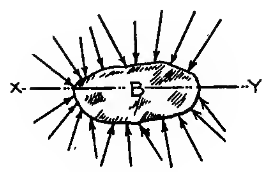

FIG. 27.

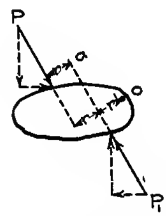

FIG. 28.

equal zero, or the body will move. But these are not the only algebraic conditions necessary when dealing with non-concurrent forces. Since the forces are not applied to a given body at a single point, ΣH and ΣV may both equal zero and still the body may rotate about a stationary axis.

For example, B in Fig. 27 represents a body acted upon by a number of forces as shown, and the lines of action of which are all in the same plane, but do not intersect in a point. Suppose P, Fig. 28, represents the resultant of the forces acting above the line xy, and P_1 the resultant of the forces acting below this line; P_1 being equal and parallel to P. Now P and P_1, it should be noted, are not in the same line of action and, since they are equal and parallel, ΣH and ΣV will both equal zero. P and P_1 will

cause rotation of the body and the amount of this rotation about any point in the same plane is measured by the force P (or P_1) multiplied by the lever arm r; that is, by the moment of a couple. Thus, it should be clear that the forces acting upon a rigid body may satisfy the conditions $\Sigma H=0$ and $\Sigma V=0$ and still the body may rotate due to the action of a couple.

The student should observe that the only way to prevent the body B from rotating about the point o, or about any other point where the body might be pivoted, would be by making P and P_1 equal and in the same line of action. Thus, we have the additional algebraic condition that is needed in order that the forces of a non-concurrent system may be in equilibrium. The condition is this—that the algebraic sum of the moments of the forces about any origin must equal zero.

Let ΣM represent the algebraic sum of the moments about any point. We may then state the three algebraic conditions necessary for equilibrium as follows:

$$\Sigma H=0. \qquad \Sigma V=0. \qquad \Sigma M=0.$$

These are the three most important equations in statics.

Problems in the equilibrium of non-concurrent forces may be solved either graphically or algebraically if the number of unknowns is not greater than three. It is easy to see algebraically that three independent equations may be written, employing the three algebraic conditions above stated, and that solving these equations simultaneously in any given case would give the three unknowns. The same number of conditions may be found graphically as will be shown in Art. 32.

The three unknowns usually desired may be classed under three general cases; namely, where the following unknowns are required: (1) point of application, direction and magnitude of one force (that is, the force is wholly unknown); (2) magnitudes of two forces and the direction of one of these forces; and (3) magnitude of three forces. The first case is nothing more than the finding of the resultant of a system of non-concurrent forces. Case (2) is treated in Art. 31 and Case (3) in Art. 56.

The magnitude and direction of the resultant of a system of non-concurrent forces (Case 1) may be found algebraically in the same manner as for concurrent forces; that is, by choosing convenient axes at right angles to each other and then resolving each force into components parallel to these axes (Art. 27). A

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point on the line of action of the resultant may be determined by finding the algebraic sum of the moments of the forces about any origin in the plane of the forces, considering positive direction plus and negative direction minus. The lever arm of the resultant, or perpendicular distance from the origin to the line of action of the force considered, may then be determined by dividing the algebraic sum of the moments by the magnitude of the resultant force.

A special case in the solution of non-concurrent forces occurs when all the forces considered are parallel. Then the number of independent equations reduces to two and it is, therefore, possible to determine but two unknowns, namely: (a) point of application and magnitude of one force; and (b) magnitude of two forces. Some explanation is required in regard to the manner of treating Case (a) algebraically. Both Case (a) and Case (b) will be treated algebraically by illustrative problems.

In order to illustrate Case (a) for parallel forces suppose P and P_1, Fig. 28, do not have the same magnitudes. The resulting tendency to motion about o is $P_1r' - P(r+r')$. If the result comes out plus, the resulting tendency to rotation is clockwise. If minus, it is counter-clockwise. It is required to find the force necessary to keep in equilibrium the forces P and P_1 applied at a distance s from the point o. It will be easily seen that

$$\frac{P_1r' - P(r+r')}{s} = \text{force required.}$$

Also, let it be required to find the distance from o that a force F must be applied in order to produce equilibrium in connection with the two given forces. It is evident that

$$\frac{P_1r' - P(r+r')}{F} = \text{distance required.}$$

Illustrative Problem.—Find the resultant of the three vertical forces shown in Fig. 29. Since the forces are all vertical, $\Sigma H = 0$ and the resultant must also act in a vertical direction. Consider downward forces positive and upward forces negative. The magnitude of the resultant may be found as follows:

$$R = 300 + 100 - 200$$
$$= 200 \text{ lb., acting down (since the result is positive).}$$

It will be noticed that a force equal and opposite to R would make the forces in equilibrium; that is, $\Sigma V = 0$, $\Sigma H = 0$, and $\Sigma M = 0$.

It is now necessary to find the point of application of the resultant R. By the point of application in this case is meant a point on the line of action of the resultant.

Take the point of application at o as shown. The algebraic sum of the moments about o is equal to $(300)\ (2)+(100)\ (8)+(200)\ (2)=1800$ ft. lb. The resulting force is 200 lb. and the problem resolves itself into finding how far from the point o the 200 lb. should be placed to have the same effect as the three loads shown or, in other words, how far away from o should a load equal and opposite to the 200 lb. resultant be placed in order to cause equilibrium. Thus, $\Sigma M=0$ may be used to find this distance.

$$\frac{1800 \text{ ft. lb.}}{200 \text{ lb.}}=9 \text{ ft. to the right of } o.$$

The student should observe that the computations would be more simple if the point x had been selected instead of the point o. It will always be found true that the work is simplified by taking the origin on the line of action of one of the forces. The computations would then be arranged as follows:

$$\frac{(300)\ (4)+(100)\ (10)}{200}=11 \text{ ft. to the right of } x.$$

Illustrative Problem.—The beam AB (Fig. 30) is 14 ft. long and loaded as shown. It is simply supported at A and C. (a) Determine the support-

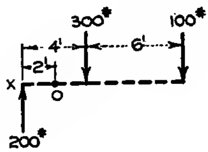

FIG. 29.

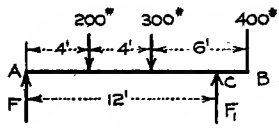

FIG. 30.

ing forces due to the three given loads. (b) Determine the supporting forces, including the weight of the beam which is 50 lb. per lin. ft.

(a) $R=200+300+400=900$ lb., acting down.

$F+F_1=R=900$ lb.

Origin at A:

$(200)\ (4)+(300)\ (8)+(14)\ (400)=12\ F_1$

$F_1=733$ lb.

$F\ =900-733=167$ lb.

$$\textit{Answers} \begin{cases} F\ =167 \text{ lb.} \\ F_1=733 \text{ lb.} \end{cases}$$

(b) Wt. of beam $=(50)\ (14)=700$ lb.

$R=900+700=1600$ lb.

$(200)\ (4)+(300)\ (8)\ +(14)\ (400)+(700)\ (7)=12\ F_1$

$F_1=1142$ lb.

$F\ =\ 1600-1142=458$ lb.

$$\textit{Answers} \begin{cases} F\ =\ 458 \text{ lb.} \\ F_1=1142 \text{ lb.} \end{cases}$$

Problem 27.—Find the resultant of the parallel forces shown in Fig. 31.

Problem 28.—The bar AB (Fig. 32) is simply supported at the ends and loaded with 1250 lb. as shown, in addition to the weight of the bar which is 4 lb. per lin. in. Find the magnitudes of the supporting forces.

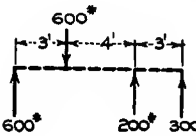

Fig. 31.

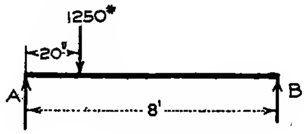

Fig. 32.

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30. The Equilibrium Polygon.—The graphical method already explained for finding a point such as n (Fig. 25) on the line of action of the resultant, cannot always be conveniently used. If the forces are parallel, or nearly so, it is not easy to obtain the intersection of the forces and, consequently, a different construction is necessary. The diagram that is used for such cases is called the equilibrium polygon. The force polygon will be needed to find the magnitude and direction of the resultant the same as before.

Consider the same four forces shown in Fig. 25. They are represented again in Fig. 33. The force polygon *abcde* for these forces is reproduced in Fig. 34. The line *ae* gives the magnitude and direction of the resultant R_3 as before. Select any point p and draw the lines *pa, pb, pc, pd* and *pe* to the vertices of the force polygon.

In the force triangle *apb, bp* and *pa* represent the magnitudes and directions of two forces P_2 and P_1 which will balance F_1 if their directions be from b to p and from p to a. Select some point o on the line of action of F_1 and draw the lines P_{2a} and P_{1a} parallel to P_2 and P_1 respectively. The force P_{2a} intersects the force F_2 at the point m. In the triangle *bpc*, forces P_3 and P_2 will hold F_2 in equilibrium if their directions be from c to p and from p to b. At the point m draw P_{3a} parallel to P_3 and produce P_{3a} (backward in this case) until it meets the force F_3 at n. In the triangle *cpd*, forces P_4 and P_3 balance the force F_3 if their directions are taken from d to p and from p to c. At the point n draw P_{4a} parallel to P_4 and produce P_{4a} (backward in this case) until it meets the force F_4 at the point k. In the triangle *dpe*, forces P_5 and P_4 balance the force F_4 if their directions are taken from e to p and from p to d. At the point k draw P_{5a} parallel to P_5, and

produce P_{5a} (backward in this case) until it meets the line of action of P_1. It is also necessary to produce P_{1a} (backward in this case). P_{5a} meets P_{1a} at the point s. It should be noticed that P_{1a} and P_{5a} are the only forces in the equilibrium polygon which are not balanced by equal and opposite forces. In the remaining

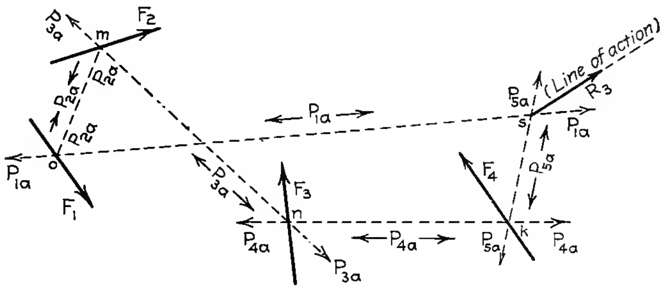

FIG. 33.

force triangle ape in Fig. 34, P_1 and P_5 will be components of the resultant R_3 if their directions be from a to p and from p to e. Since the lines of action of P_1 and P_5 are shown by P_{1a} and P_{5a}, the resultant must pass through the point s where they intersect.

The equilibrium polygon is more generally used and is much to be preferred over that of the method of producing forces until

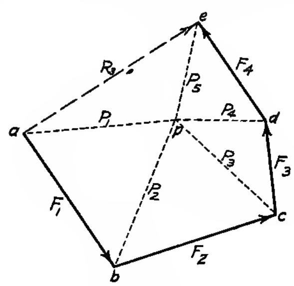

FIG. 34.

they intersect. It is very seldom that the latter construction comes conveniently within the limits of a drawing.

The point p in Fig. 34 is called the *pole*; the lines pa, pb, pc, etc., are called *rays*; and the lines om, mn, nk, etc., in Fig. 33, are called *strings*.

Since p is any point that may be selected, it should be taken so that it will be most convenient for the solution of the given

problem. It should be remembered that the magnitude and direction of the resultant of any number of non-concurrent forces is given by the force polygon and a point on its line of action by the equilibrium polygon. The force polygon must first be drawn and the resultant determined in both magnitude and direction by the closing side. The pole p should next be selected and the rays drawn, to which the strings of the equilibrium polygon should be made respectively parallel. The line through the intersection of the first and last strings parallel to the direction of the resultant in the force polygon is the line of action of the resultant.

If the force R_3 acted in the opposite direction, the system would be in equilibrium and the forces would follow in order around the force polygon. The system spoken of in equilibrium would then be forces F_1, F_2, F_3, and F_4 and a force equal and opposite to R_3 acting through the point s. If a force equal and opposite to R_3 should accidentally be placed to one side or the other of the point s, but still parallel to its direction as shown by the force polygon, the intersection of P_1 and P_5 would not fall on its line of action. We would then say that the equilibrium polygon did not close. Thus, it is easily seen for a given system of forces that, even if the force polygon closes, the equilibrium polygon may not close.

When the force polygon closes and the equilibrium polygon does not, the result is that of a couple. For such a case the resultant of the forces F_1, F_2, F_3 and F_4 would not be in the same line of action as the remaining force which is equal and opposite to R_3, and equilibrium could not result. Equilibrium exists when the moment of the couple is zero.

The graphical condition of equilibrium for several forces not meeting at the same point may now be explained by saying that both the force and equilibrium polygons must close. If the former closes and the latter does not, the given forces are not in equilibrium.

Problem 29.—Transfer the forces shown in Fig. 33 (in magnitude, line of action, and direction) to a blank sheet of paper. Represent the force F_2 equal, but opposite in direction, to the way it is shown. Determine the magnitude, direction, and line of action of the resultant of the given system of forces by means of the force and equilibrium polygons.

ASSIGNMENT 5

CHAPTER IV

REACTIONS

[The principles of statics have now been explained both algebraically and graphically. Also, the student should know how to determine the live and dead loads on structures. The next computations in design should be for the purpose of finding reactions and then all the outer forces may be completely determined. These reactions may be found by the preceding principles of statics.]

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31. Structures Statically Determinate with Respect to the Outer Forces.—A structure is *statically determinate* when both outer and inner forces may be determined by the aid of statics; *statically indeterminate* when the reverse is true. If all the outer forces may be found by statics the structure is said to be *statically determinate with respect to the outer forces* whether or not it is

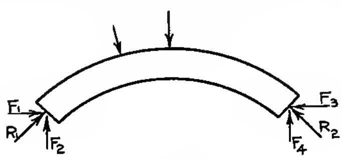

FIG. 35.

possible to determine the inner forces by the same means. Referring to Fig. 35, it will be seen that six conditions are needed in order to completely determine the two reactions R_1 and R_2; namely, their points of application, their directions (meaning the angle with the horizontal), and their magnitudes.

In such structures as roof and bridge trusses, the loads and reactions all act in the same vertical plane and are in equilibrium; consequently, when solving algebraically, the three equations of statics may be used; that is, $\Sigma H=0$, $\Sigma V=0$, $\Sigma M=0$. The student should already know, from the study of simultaneous

equations in algebra, that each independent equation furnishes the means of finding one unknown quantity. Thus, from our three equations of statics we may obtain three of the conditions above mentioned. The other three conditions should be determined from the way in which the structure is supported. If a structure is so supported that three conditions are given in addition to the three equations of statics, then the structure is called statically determinate with respect to the outer forces. If this is not true the structure is called statically *indeterminate* with respect to the outer forces.

If there are less than three unknown conditions in regard to the manner in which a structure is supported, then the structure is in general unstable and will tend to move bodily under the applied loads. For example, suppose the supporting forces to have only their magnitudes unknown. Then unless the resultant of these reactions is in the same line of action as the resultant of the applied loads, equilibrium cannot exist. The structure, therefore, will move and is termed unstable.

The three conditions generally given by the manner in which a structure is supported are the points of support and the direction of one of the reactions. One end of a structure placed on rollers (which are round bars of iron or steel) makes the reaction at that end at right angles to the supporting surface since the rollers, if in

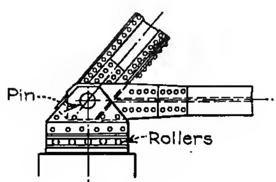

FIG. 36.

good condition, cannot offer resistance to motion along this surface. If a point on the line of action of this supporting force is known and if the other point of support is definitely fixed, the structure becomes statically determinate with respect to the outer forces.

A student should understand that if a structure is hinged at a support, the line of action of the force at that support passes through the hinge. A hinge generally is a steel cylindrical shape of short length and but a few inches in diameter and called

a *pin*. When employed at a support it rests upon a shoe which in turn rests upon the support.

When a hinge is placed at the same support where rollers are used (Fig. 36) the reaction is at once determined in direction and point of application.

We have seen that rollers cause a reaction to act at right angles to the supporting surface. They also serve the purpose of allowing steel structures to expand and contract with changes in temperature of the steel and thus prevent additional stresses in the different members.

Problem 30.—The loads, spans, and all other data in Figs. 37, 38, and 39 are assumed to be known. Which of these structures are statically determinate with respect to the outer forces? Give reasons.

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32. Determination of Reactions (*Algebraic and Graphical*).—Reaction problems when solved algebraically will generally be simplified by finding the horizontal and vertical components of the reactions and then obtaining the magnitude of either reaction

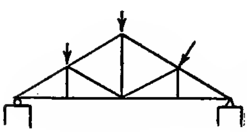


Fig. 37.

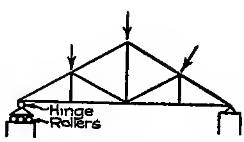


Fig. 38.

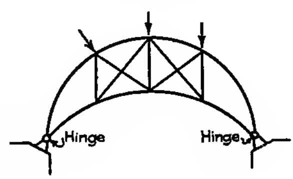


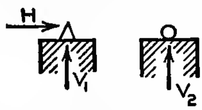
Fig. 39.

by computing the square root of the sums of the squares of its two components. With one end on rollers and resting upon a horizontal surface, the vertical component at that support is the reaction required, and the horizontal component is zero. With a roller end resting upon an inclined surface, the reaction at that support will have both a vertical and a horizontal component, but there is at once a relation between them due to the fact that the reaction must act at right angles to the supporting surface. If one component is found the other component can be at once determined without resorting further to our three equations of statics.

If a load be distributed over a considerable area, as wind pressure for example, instead of being applied at a point, the resultant

of this load may be used in the computations as a concentrated load.

Structures supported at one end by a tie-rod should be considered as having the reaction at that point fixed in direction. A tie-rod is incapable of carrying compression or bending, and thus the reaction which it carries must act along its axis and produce tension in the rod.

In all the following problems, the reaction at points shown thus  may be considered to have both a horizontal and vertical component. At points shown thus  the reaction is to be taken as acting at right angles to the supporting surface. When solving algebraically, indicate the horizontal and vertical components of reactions at each support and their assumed directions, thus 

and find these components only in each algebraic problem. If the result comes out negative, the force acts in the opposite direction to that assumed.

It is seldom found in practice that the point of application of a reaction is definitely fixed. For short beams which deflect but little and which rest at the ends upon steel-bearing plates (inserted in order to distribute the load over the masonry supports), it is sufficiently exact to consider the reaction as applied at the center of bearing, but this assumption is by no means an exact one. For long girders, especially, the deflection would be so great that the center of bearing would be brought near the edge of support and the assumption would not hold. However, if a pin bearing is used with rollers a uniform bearing on the support is ensured. The reaction is then considered to pass through the pin center, but this will not be quite true if the pin is badly turned or the bearing surface of the shoe upon which it rests imperfect.

The symbol which will be used for a fixed end is not intended to represent a knife bearing, but simply means that the point of application is determined and that the reaction may act in any direction; that is, up, down, horizontal, or inclined.

Illustrative Problem.—A beam is loaded as shown in Fig. 40. Find the reaction at *A* and *B* by the algebraic and graphical methods. Neglect weight of beam itself.

Algebraic Method

$\Sigma H = 0 \therefore H_1 = 0$
$\Sigma M = 0$ Origin at A.
$(6)(6) + (20)(22.5) - 15V_2 - (10)(5) = 0$
$V_2 = 29.1$ tons, acting up, since result is positive.

$\Sigma V = 0 \quad 10 + 6 + 20 - 29.1 = V_1$
$V_1 = 6.9$ tons, acting up.

(If a check on V_1 is desired, it may be obtained by applying $\Sigma M = 0$ about B as an origin.)

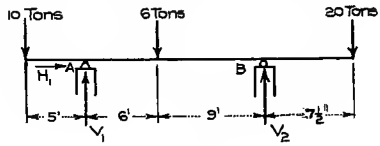


Fig. 40.

Graphical Method

In Fig. 41, the force polygon is drawn for the given forces. The forces are designated by letters instead of by weight. It can easily be seen that $H_1 = 0$ or the forces would not be in equilibrium. The force polygon, consequently, becomes a straight line since the forces are all vertical. $ab = F$, $bc = F_1$, $cd = F_2$, $de = V_2$, $ea = V_1$. It is not possible to determine the point e until after the equilibrium polygon is drawn. The construction should be

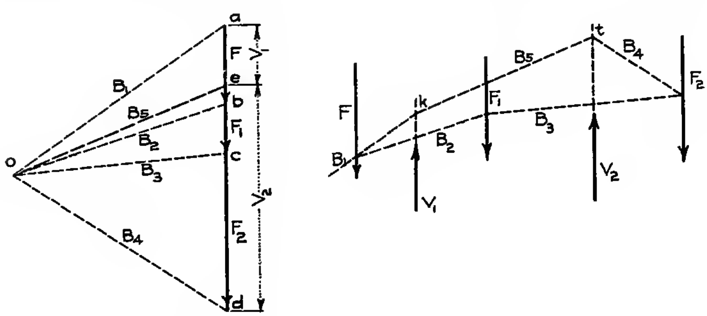


Fig. 41.

familiar to the student. The string B_4 intersects V_2 at t. The string B_1 intersects V_1 at k. The line oe in the force polygon drawn parallel to kt in the equilibrium polygon divides the line ad into two parts, de and ea, which represent V_2 and V_1 respectively. kt is drawn in the equilibrium polygon because we know the forces are in equilibrium and the equilibrium polygon should close. B_4 and B_5 will be seen to balance V_2, also B_1 and B_5 balance V_1.

Problem 31.—Find the horizontal and vertical components of the reactions A and B, Fig. 42, by the algebraic method. (Considerable labor may

be saved by resolving the inclined forces into horizontal and vertical components. The lever arm of the horizontal components about either support is zero, leaving only the vertical components to be considered when applying $\Sigma M = 0$.)

Problem 32.—Find reactions C and D, Fig. 43, by the algebraic method.

Problem 33.—Find reactions E and F, Fig. 44, by the algebraic method.

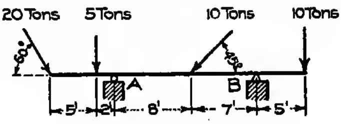


FIG. 42.

We have seen that in order to use only the force polygon in the graphical method not more than two conditions should be unknown. We know that in order to find the reactions of statically determinate structures three conditions must be supplied in addition to the three which are given by the manner in which the structure is supported. Thus, when the graphical

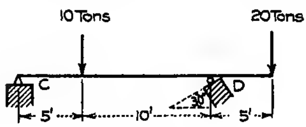


FIG. 43.

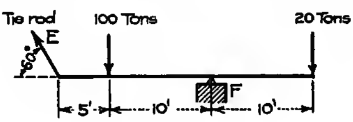


FIG. 44.

method is used one condition must be determined either by means of one of our three algebraic equations, by means of the intersection method, or by constructing the equilibrium polygon. The other two conditions can then be found by the force polygon.

Problem 34.—In problem 31 find the reaction at A algebraically and determine the other two conditions by the force polygon.

Problem 35.—Change the 10 ton load of problem 32 to 25 tons and solve by method described in problem 34. (Vertical component at D will determine one condition since the relation between the inclined reaction and vertical component is known.)

Problem 36.—Substitute a 40 ton load for the 20 tons given in problem 33 and solve in the same manner as for problems 34 and 35.

Illustrative Problem.—Find the horizontal and vertical components of the reactions A and B, Fig. 45a, for the wind load assumed. Solve by both the algebraic and graphical methods.

Only the resultant wind pressure need be considered in this problem and it will act at the center of AC.

Algebraic Method

Let us consider the wind pressure acting on a strip of roof surface having a length AC and a width of one foot.

Normal pressure = (20) $(AC) = P_n$. Denote horizontal and vertical components of P_n by H_x and V_x respectively.

$$\frac{H_x}{P_n} = \frac{12}{AC}$$

$$H_x = \frac{12\ (P_n)}{AC} = (12)\ (20)$$

Similarly, $V_x = (25)\ (20)$

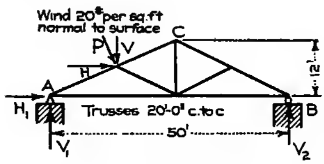


Fig. 45a.

Thus, from the above it should be clear that we can obtain these H_x and V_x components by multiplying the normal pressure in lb. per sq. ft. by the projection of the upper chord (AC in this case) on a plane at right angles to the direction of the desired component.

Since the trusses are $20'-0''$ center to center the H and V components of the total normal pressure P acting on the truss are as follows:

$$H = H_x\ (20) = (12)\ (20)\ (20) = 4800 \text{ lb.}$$
$$V = V_x\ (20) = (25)\ (20)\ (20) = 10{,}000 \text{ lb.}$$

Origin at A

$$(10{,}000)\left(\frac{25}{2}\right) + (4800)\ (6) - 50V_2 = 0.$$

$V_2 = 3080$ lb., acting up.

$3080 + V_1 - 10{,}000 = 0.$

$V_1 = 6920$ lb., acting up.

$4800 + H_1 = 0.$

$H_1 = 4800$ lb. $(-)$, acting toward the left.

Graphical Method

Fig. 45b shows how the reactions are obtained by means of the force and equilibrium polygons. The construction is as follows: Draw P in force polygon. Choose pole o. Draw rays B_1 and B_2. Draw strings B_1 and B_2 so that B_1 passes through the point of support A, A being a known point in

the line of action of R_1. Draw string B_3 from A through the point m which is the intersection of string B_2 and the reaction V_2. Draw ray B_3 in force polygon. Knowing V_2 to be vertical its magnitude is easily determined. R_1 is the closing side of the force polygon in magnitude and direction. Draw a line through A parallel to R_1 of the force polygon, thus giving the line of action of the left reaction. Lines ab and bc in the force polygon represent the H and V components of R_1 respectively. It is seldom necessary in graphical work to determine the components of reactions.

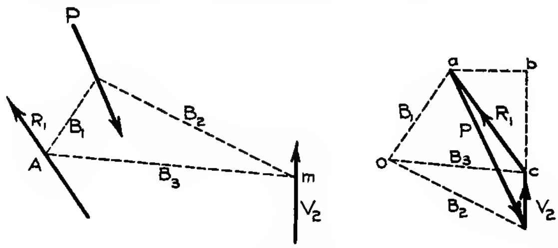


FIG. 45b.

Fig. 45c shows how the reactions are obtained by producing the forces until they intersect.

For this particular example the intersection method is more simple, but this is not always so.

The student should observe that the equilibrium polygon gives the magnitude of V_2, while the latter method determines the direction of R_1. The statement made previously should now be clear; namely, that one condition must be determined either by the equilibrium polygon or by the intersection method, or by an algebraic equation, before the force polygon can be used to determine the other two.

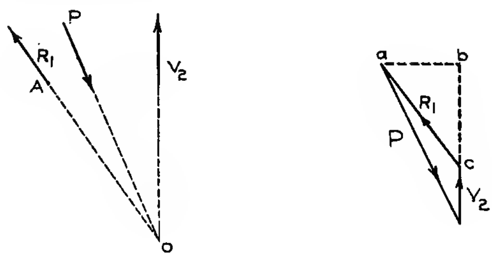


FIG. 45c.

Illustrative Problem.—Find the reactions A and B, Fig. 46a, by the algebraic method and also graphically by means of the force and equilibrium polygons.

ALGEBRAIC METHOD

$$V = (20)\ (30)\ (20) = 12{,}000 \text{ lb.}$$
$$H = (20)\ (15)\ (20) = 6000 \text{ lb.}$$
$$H' = (5)\ (20)\ (50) = 5000 \text{ lb.}$$

Origin at A.

$$(5000)\frac{5}{2}+(6000)\ \frac{25}{2}+12{,}000\ (5)-30V_1=0$$

$V_1 =$ 4920 lb., acting up.

$4920+V_2-12{,}000=0.$

$V_2 =$ 7080 lb., acting up.

$6000+\ 5000+H_1=0.$

$H_1=11{,}000$ lb. $(-)$, acting toward the left.

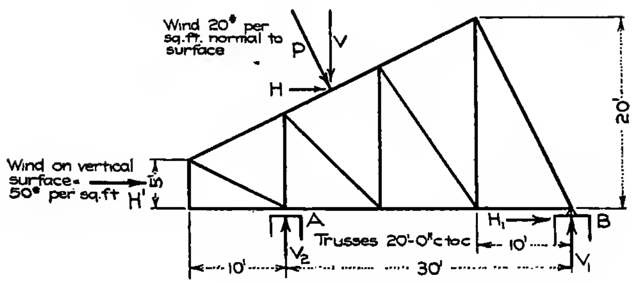


Fig. 46a.

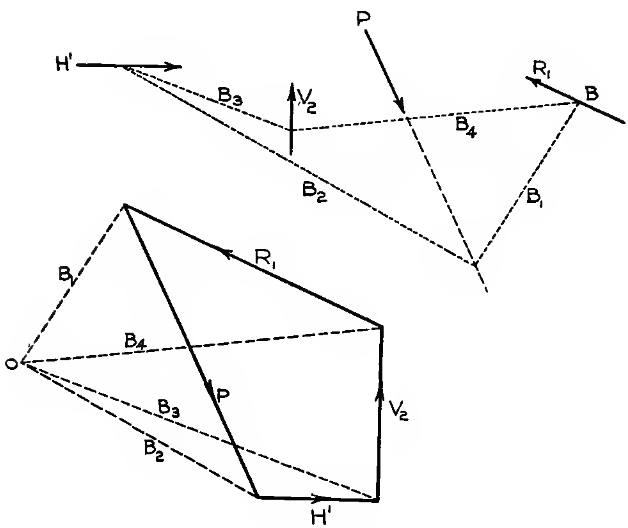


Fig. 46b.

Graphical Method

Fig. 46b shows how the reactions are obtained by means of the force and equilibrium polygons.

Problem 37.—Same truss as in preceding illustrative problem, but with wind blowing from right, 40 lb. per sq. ft. normal to inclined surface.

Problem 38.—Find the reactions E and F, Fig. 47. Both methods.

Problem 39.—Find the reactions G and H, Fig. 48. Both methods.

Problem 40.—Find the reactions K and L, Fig. 49, by the algebraic

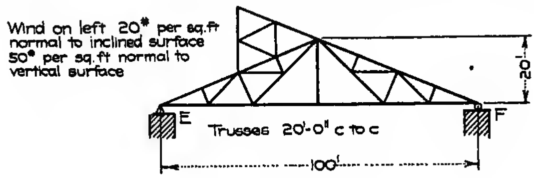


FIG. 47.

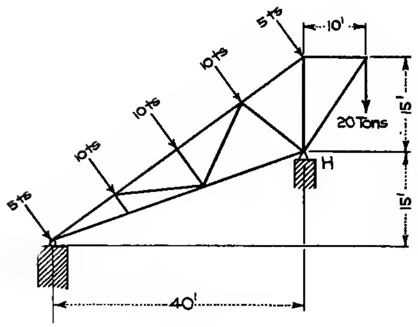


FIG. 48.

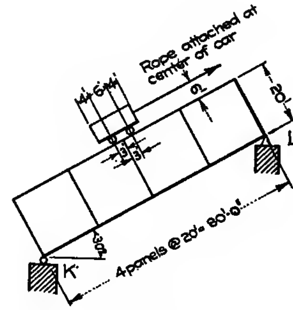


FIG. 49.

method. Weight of car = 10 tons. Car is held in equilibrium by rope. Neglect friction between car and truss.

Problem 41.—The skeleton outline of a steel roof truss is shown in Fig. 50. Determine reactions algebraically with the wind upon the fixed side. Use tables based upon Hutton's formula for wind pressure on inclined surfaces.

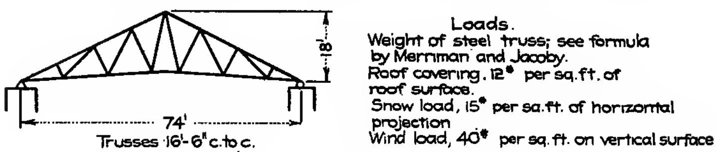


FIG. 50.

Roof trusses of short span are generally fixed at both ends to the walls of the building, thus becoming statically indeterminate with respect to the outer forces. In this case the reactions for the wind load are determined separately from those caused by

the dead and snow loads. Dead and snow loads cause only vertical reactions. The wind load causes the reactions to be inclined and the horizontal components tend to overturn the walls of the building. One of two assumptions is usually made,

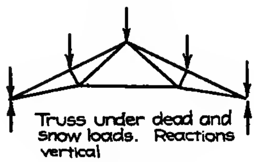


FIG. 51

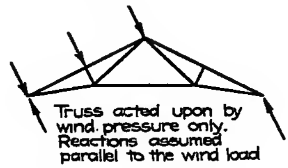


FIG. 52.

either (a) that the horizontal components of the two wind reactions are equal, or (b) that the direction of the wind reactions are parallel to the resultant wind load. (Figs. 51 and 52).

Problem 42.—Suppose the roof truss shown in Fig. 50 to be firmly fixed at the ends. Assume wind load reactions parallel to the resultant wind load. Determine reactions algebraically.

Illustrative Problem.—Fig. 53 represents a Howe bridge truss of 120 ft. span, with 12 equal panels. Neglecting the dead load on the end panel points, determine the reactions algebraically for a dead load of 9000 lb. on each intermediate panel point and a live load of 20,000 lb. on panel points marked *a*, *b*, and *c*.

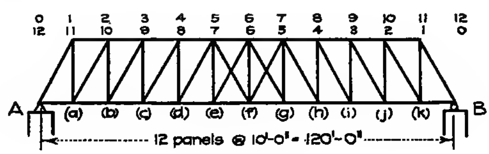


FIG. 53.

Reactions A and B are both vertical since the loads are vertical, which is generally the case on bridge trusses. Then again, since the panels are all equal the algebraic method is by far the more convenient one to use. The stringers at each end either rest directly upon the abutments or upon end floor beams. In either case the load on an end panel point is fully carried by the support beneath, thus causing no reaction at the other support and hence no stresses in the truss. This is the reason for the omission of the dead load on the end panel points in this problem. In designing the details at A and B, however, the loads at these points must be considered.

Reactions A and B each receive one-half the dead load, or $9000 \times 5\ 1/2 =$ 49,500 lb.

Reaction A for the live load is

$$\frac{(90)\ (20{,}000) + (100)\ (20{,}000) + (110)\ (20{,}000)}{120} \text{ (Origin at } B)$$

$$= \frac{(20{,}000)\ \ (90+100+110)}{120} = \frac{(20{,}000)\ \ (9+10+11)}{12} = 50{,}000 \text{ lb.}$$

This may be more conveniently calculated by obtaining the last equation directly, which means that we take the panel as a unit of length. Thus, the B reaction for the live load is

$$(20{,}000)\ \frac{(1+2+3)}{12} = 10{,}000 \text{ lb.} \quad (\text{Origin at } A)$$

Total reaction $A = 49{,}500 + 50{,}000 = 99{,}500$ lb.
Total reaction $B = 49{,}500 + 10{,}000 = 59{,}500$ lb.

Problem 43.—Suppose a dead load of 10,500 lb. acts on each intermediate panel point of the bridge shown in Fig. 53. Also a live load of 18,200 lb. acts at panel points b, c, d, and e. Determine the reactions algebraically.

ASSIGNMENT 6

CHAPTER V

SHEAR AND MOMENT

[The student should now be able to completely determine the outer forces acting upon structures of the type discussed. There are two very important functions of the outer forces; namely, shear and moment. A knowledge of the methods used in determining these functions is necessary before any attempt should be made to find the inner forces. Then after once the inner forces are known a structure can be proportioned. Although the student should be acquainted with shear and moment from the course in "Strength of Materials," nevertheless, for clearness in this study, the meaning of these two functions of the outer forces will be briefly restated. In dealing with the subject of shear and moment, coplanar forces only need be considered, as previously mentioned.]

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33. Shear.—Consider the forces acting on a beam to be resolved into horizontal and vertical components. Then the shear at any section is the algebraic sum of the vertical forces acting on either side of the section and is the force by which the part of the beam on one side of the section tends to slide by the part on the other side. This tendency is opposed by the resistance of the material to transverse shearing.

When the resultant force acts upward on the left of the section the shear is called *positive* and when it acts downward on the same side of the section it is called *negative*. Since $\Sigma V=0$ when we consider the forces on both sides of the section, then the resultant of the forces on the right of the section must be equal and opposite in direction to the resultant of the forces on the left of the section. Thus, it makes no difference which side of the section we consider, the shear is *positive* when the resultant on the left is upward and when the resultant on the right is

downward. Also the shear is *negative* when the resultant on the left is downward and when the resultant on the right is upward.

At the section *ab*, Fig. 54, the shear, since there are no loads between the section and the left support, equals the left reaction and is positive. This is also true of any section between the left support and the section *cd*. The shear to the right of *cd* is negative and is equal to the right hand reaction.

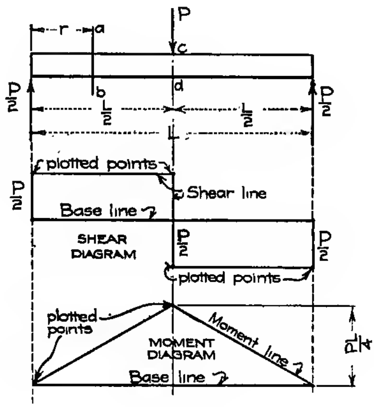

FIG. 54.

34. Bending Moment.—The bending moment (or moment) at any section of a beam is the algebraic sum of the moments of the forces acting on either side of the section about an axis through the center of gravity of the section and is the moment which measures the tendency of the outer forces to cause the portion of the beam lying on one side of the section to rotate about the section. This tendency to bend the beam is opposed by internal fiber stresses of tension and compression.

When the resultant moment on the left of the section is clockwise the moment is called *positive* and when it is counter-clockwise on the same side of the section it is called *negative*. Since $\Sigma M=0$, when we consider the forces on both sides of the section, then the resultant moment of the forces on the left of the section is equal and opposite to the resultant moment of the forces on the right of the section. Thus, it makes no difference which side of the section we consider, the moment is *positive* when the resultant moment of the forces on the left is clockwise and when the resultant moment of the forces on the right is counter-clockwise.

Also, the moment is *negative* when the resultant moment of the forces on the left is counter-clockwise and when the resultant moment of the forces on the right is clockwise.

At the section *ab*, Fig. 54, the moment is $\frac{P}{2}(r)$. It increases uniformly from the left support where it is zero to the section *cd* where is it $\left(\frac{P}{2}\right)\left(\frac{L}{2}\right)=\frac{PL}{4}$

Positive bending moment causes compression in the upper fibers of a beam supported at its two ends and tension in the lower fibers. The reverse is true for negative bending moment.

35. Shear and Moment Diagrams.—The variation in the shear or bending moment from section to section for fixed loads may

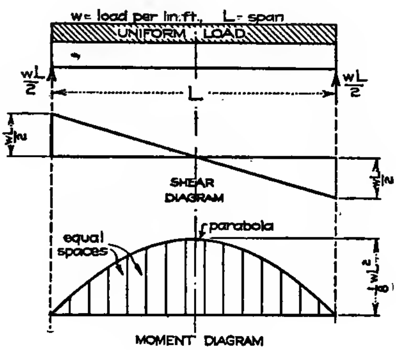

Fig. 55.

be well represented by means of diagrams, called shear and moment diagrams. The diagrams are constructed by laying off a *base-line* equal to the length of the beam and marking off on this line the positions of the loads and the reactions. Positive shear and moment at given points should be represented above the base-line and negative shear or moment below this line. Points are plotted vertically above or below given points on the base-line and the distance these plotted points are from the base-line should represent to some scale the magnitude of the shear or moment at these given points on the beam. The line joining the points plotted in this way is called the shear or moment line, depending upon whether a shear or moment diagram is being drawn.

To illustrate, in Fig. 58, the ordinate *ab* represents the value of the shear at the point *b* of the beam and the ordinate *cd* represents the value of the moment at the point *d*.

In shear diagrams for uniform loading, ordinates need only be erected at the ends of the beam and at the points of support. If

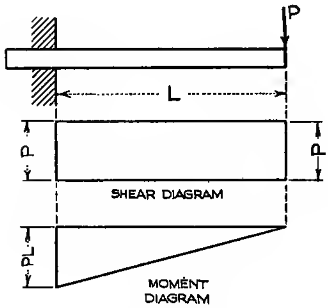

FIG. 56.

concentrated loads are also applied to the beam, ordinates must also be plotted at their points of application.

In moment diagrams for uniform loading, ordinates should be erected and points plotted at the reactions and every foot or two along the beam. If concentrated loads are also applied to the

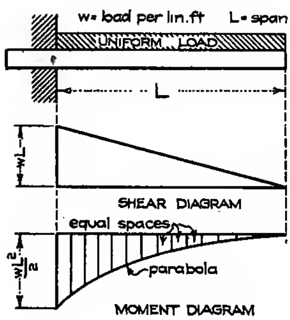

FIG. 57.

beam, ordinates must also be plotted at their points of application.

If the shear or moment lines are not completely determined by the above rules, additional points should be taken.

A *cantilever* beam is a beam having one end fixed and the other end free. The reaction at the fixed end is indeterminate, but the

shear or bending moment at a given section may be easily found by considering the loads between the section and the free end. The reaction at the fixed end is not needed.

The student should study carefully the shear diagrams shown

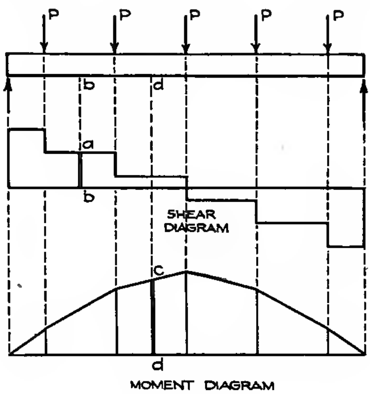

Fig. 58.

in Figs. 54, 55, 56, 57, 58, and 59. In all cases the weight of the beam is neglected.

36. Maximum Shear.—It is always desirable in proportioning beams to know the greatest or maximum value of the shear in a

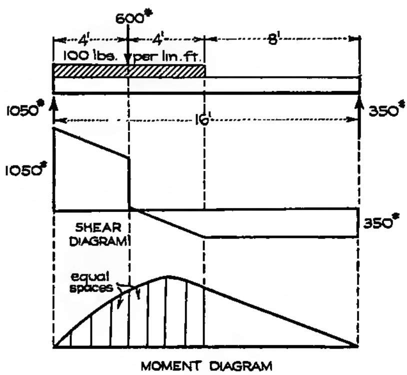

Fig. 59.

given case. The student should examine all the shear diagrams referred to above and the following statements will be found true.

1. In cantilevers fixed in a wall, the maximum shear occurs at the wall.

2. In simple beams, the maximum shear occurs at the section next to one of the supports.

37. Maximum Moment.—If the student will compare the corresponding shear and moment diagrams, he will see that the maximum moment in a simple beam occurs where the shear changes sign; that is, where the shear line crosses the base-line. This could also be shown algebraically.

By the help of this principle we need only to construct the shear line and observe from it where the shear changes sign; then compute the bending moment for that section.

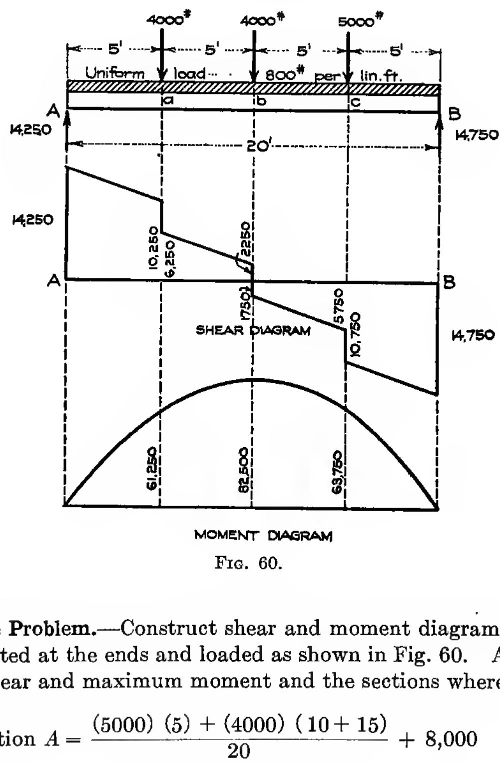

FIG. 60.

Illustrative Problem.—Construct shear and moment diagrams for a 20 ft. beam supported at the ends and loaded as shown in Fig. 60. Also, find the maximum shear and maximum moment and the sections where they occur.

$$\text{Reaction } A = \frac{(5000)\ (5) + (4000)\ (10 + 15)}{20} + 8{,}000$$

$$= \frac{(5000)\ +\ (4000)\ (5)}{4} + 8{,}000$$

$$= 14{,}250 \text{ lb.}$$

$$\text{Reaction } B = 13{,}000 + 16{,}000 - 14{,}250$$

$$= 14{,}750 \text{ lb.}$$

Shear at $A = 0$

Shear at section just to right of $A = 14{,}250$

$$\text{Shear at } a \begin{cases} \text{to left} = 14{,}250 - (800)\ (5) = 10{,}250 \\ \text{to right} = 10{,}250 - 4000 = 6250 \end{cases}$$

$$\text{Shear at } b \begin{cases} \text{to left} = 6250 - (800)\ (5) = 2250 \\ \text{to right} = 2250 - 4000 = -1750 \end{cases}$$

$$\text{Shear at } c \begin{cases} \text{to left} = -1750 - (800)\ (5) = -5750 \\ \text{to right} = -5750 - 5000 = -10{,}750 \end{cases}$$

Shear at section just to left of $B = -14{,}750$

$$-10{,}750 - (800)\ (5) = -14{,}750 \text{ (check)}$$

Shear at $B = 0$

We shall only determine the moment at points A, a, b, c and B. Moments should also be found at sections 2 ft. apart on this beam to completely determine the moment curve.

Moment at $A = 0$

$$\text{Moment at } a = (14{,}250)\ (5) - (800)\ (5) \left(\frac{5}{2}\right) = 61{,}250$$

$$\text{Moment at } b = (14{,}250)\ (10) - (8000 + 4000)\ (5) = 82{,}500$$

$$\text{Moment at } c = (14{,}750)\ (5) - (800)\ (5) \left(\frac{5}{2}\right) = 63{,}750$$

Moment at $B = 0$

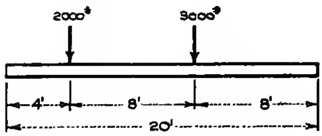

FIG. 61.

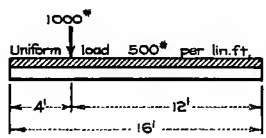

FIG. 62.

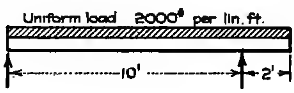

FIG. 63.

The maximum shear $= -14{,}750$ lb. at a section just to the left of the right support.

The shear changes sign at section b, consequently, the moment is a maximum at that point $= 82{,}500$ ft. lb.

Note.—In some cases the shear does not change sign at the point of application of a concentrated load and in such a case the position of the section, where the bending moment is a maximum, must be scaled or computed from the shear diagram to the nearest one-tenth of a foot.

Problem 44.—In Fig. 61 find the maximum shear and the maximum moment neglecting the weight of the beam. Also construct the shear and moment diagrams.

Problem 45.—A beam as shown in Fig. 62 is loaded with a uniformly distributed load of 500 lb. per ft. including weight of beam and a load of 1000 lb. at 4 ft. from left support. Determine maximum shear and maximum moment, and construct shear and moment diagrams.

Problem 46.—Solve problem 45 substituting 5000 lb. in place of the 1000 lb. concentrated load.

Problem 47.—Determine maximum shear and maximum moment on the beam shown in Fig. 63. Construct diagrams.

---

38. Moment Determined Graphically.—The bending moment at any section of a beam due to concentrated vertical loads may readily be determined by means of the force and equilibrium polygons. Construct the force polygon (Fig. 64a) by drawing the line *ad* to represent the sum of the loads P_1, P_2, and P_3 on a

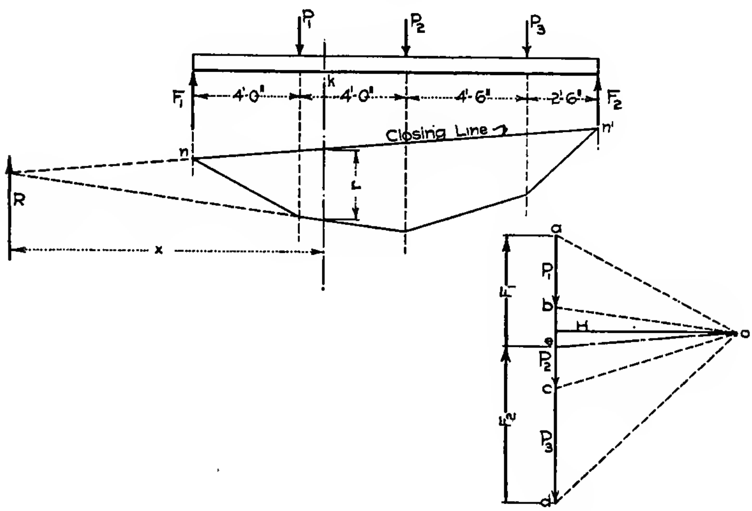

FIG. 64a.

suitable scale, and divide *ad* into segments *ab*, *bc*, and *cd*, representing respectively the given loads P_1, P_2, and P_3.

Choose a convenient pole *o*, such that the distance from *o* perpendicular to *ad* (called *pole-distance*) represents on the same scale an even number of units of force. The pole-distance is marked *H*.

Draw the rays *oa*, *ob*, *oc*, and *od*. Construct the equilibrium polygon and draw *oe* in the force polygon parallel to the closing side *nn'*, thus determining the point *e*. Then *de* and *ea* are the

two reactions F_2 and F_1 respectively, which acting upwards close the force polygon.

Let the bending moment M be required at any section of the beam such as k. Draw a vertical line through the section, cutting two sides of the equilibrium polygon, and let the ordinate intercepted between these sides be called r. The intersection of these sides produced gives the point of application of the resultant of the forces P_1 and F_1, the magnitude of which is represented by eb in the force polygon; that is, $F_1 - P_1 = ae - ab = eb$. It should

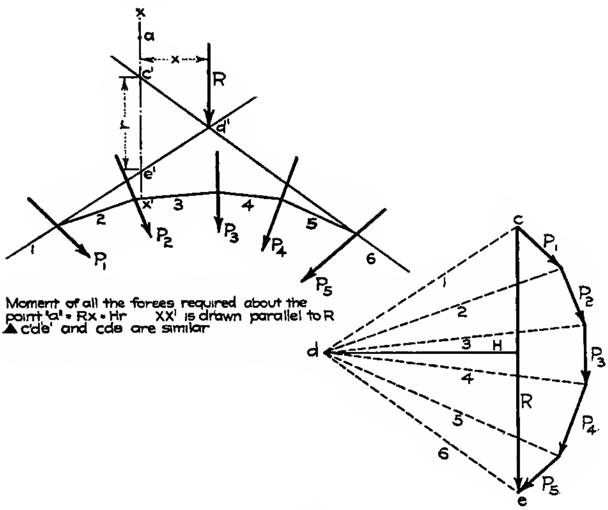

FIG. 64b.

be noticed that F_1 and P_1 act in opposite directions and, consequently, the resultant of these two forces is their difference. Let this resultant be called R and its horizontal distance from k equal x. Then, $M = Rx$.

The triangle obe is similar to the triangle which has a base r and an altitude x (sides respectively parallel) and, since eb is equal to R, we have

$$\frac{x}{H} = \frac{r}{R} \text{ or } Rx = Hr.$$

Therefore, the bending moment of the forces on the left of the section is

$$M = Hr.$$

Since H is constant the bending moment at any point in the

span is proportional to the vertical ordinate of the equilibrium polygon at that point.

Suppose in the equilibrium polygon 1/4 in. = 1 ft., and $H =$ 2000 lb., then 1/4 in. in the equilibrium polygon represents 2000 ft. lb. That is, each inch on the vertical ordinate of the equilibrium polygon represents $2000 \times 4 = 8000$ ft. lb. of bending moment. For instance, if a vertical ordinate at a given section scales 2.45 in. the bending moment of that section under the above conditions is $8000 \times 2.45 = 19{,}600$ ft. lb.

Remember, that in the above method of finding bending moment, H is constant only when the loads are all vertical. The method may be generalized and applied to loads in any direction. (Fig. 64b).

The graphical representation of bending moment at every point in the span can be applied to cases of uniform loading, but the construction is difficult and the algebraic method is much more simple. When a beam is subjected to both uniform and concentrated loads it is sometimes convenient to find the bending moment for the concentrated loads by the graphical method, and the bending moment for the uniform load by the algebraic method. The algebraic sum of the two moments at any given section will give the correct moment at that section.

Problem 48.—Find the bending moment graphically at the point of application of the 3000 lb. load in Fig. 61.

Problem 49.—In Fig. 64a let $P_1 = 2500$ lb., $P_2 = 4000$ lb., and $P_3 = 1250$ lb. Find the bending moment graphically at the points of application of the three loads.

ASSIGNMENT 7

CHAPTER V—*Continued*

39. Effect of Floor Beams.—The method just explained of determining shear and moment at any section of a simple beam caused by loads which are fixed, may also be employed in finding shear and moment on girders and trusses for a given position of the live load. Where floor beams are used, however, the girders (or trusses) receive the floor load only at the panel points and the student should have clearly in mind the manner in which the loads coming upon the floor are transmitted to these panel points before attempting to compute the resulting shears and moments.

Floor beams are ordinarily riveted to the sides of girders as in a through plate girder bridge (Fig. 8). For clearness in presentation, however, the floor beams will be shown as resting upon the girders and the stringers upon the floor beams (Fig. 66). The shears and moments are identical for the two cases. Girders are usually placed parallel to each other and any load coming upon the planking or rails (or whatever the flooring may be) is transmitted by means of the stringers to the floor beams and thence to the girders, each girder receiving a proportional part. The loads given in each case will be the proportional part of the total load considered which is actually transmitted to the given girder.

Let F be the proportional part of an applied load which is transmitted to a given girder. As shown in Figs. 65 and 66 it will be transmitted at panel points 2 and 3. Panel point 3 will receive $F\frac{a}{p}$ and panel point 2 will receive $F\frac{(p-a)}{p}$ or, in other words, these panel points receive the reactions of a simple beam one panel in length, the stringers not being continuous over the floor beams.

In Fig. 66 considering only the applied load shown, the left hand reaction R_1 equals $F\frac{a+b}{L}$ and the right hand reaction R_2 equals $F\frac{L-(a+b)}{L}$, the same as if there were no floor beams.

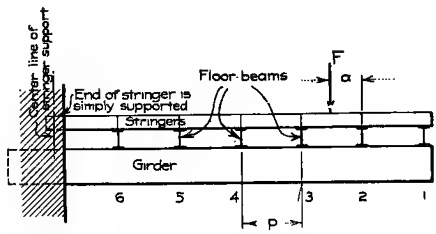

FIG. 65.

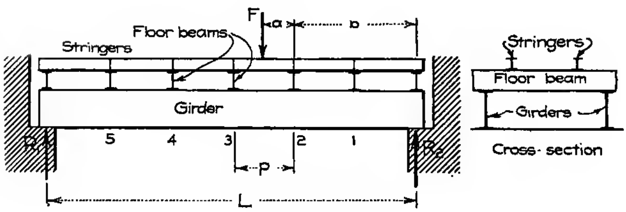

FIG. 66.

To prove this, it is only necessary to distribute a proportional part of the load F to the panel point 3 and also the proper amount to the panel point 2 and determine the reactions.

$$\text{Load at } 3 = F\,\frac{a}{p}$$

$$\text{Load at } 2 = F\,\frac{(p-a)}{p}$$

$$\text{Left hand reaction } = \frac{F\frac{a}{p}(b+p) + F\,\frac{(p-a)}{p}\,b}{L}$$

$$= F\frac{(a+b)}{L} \text{ (same as without floor beams).}$$

$$\text{Right hand reaction} = F - \frac{F(a+b)}{L}$$

$$= F\frac{L-(a+b)}{L} \text{ (same as without floor beams).}$$

The student must keep in mind the fact that we are discussing the distribution of loads on structures requiring floor beams as a means to a clear understanding of the way in which shear and moment varies in such structures and the method of computing these functions. It should be clear from "Strength of Materials" that it is necessary to ascertain the maximum moment and the maximum shear at given points in simple beams in order to design them correctly. This is also true of trusses in general and especially so of bridge trusses. Loads are applied to bridge trusses only at floor beams and the moment and shear are required at these points.

In through plate girder and truss railroad bridges, the stringers and rails are generally equally spaced about the center line between girders or trusses. If the bridge is single-track, a girder (or truss) thus receives one-half the weight of train; that is, the load coming upon one rail (Fig. 6). The above discussion applies directly to such a case, the load F being any wheel load which may come upon one rail.

The student should prove the following facts to his own satisfaction. The first four statements refer to a girder supported at one or both of its ends. Statements 5 and 6 explain themselves. The load considered is the proportional part of the floor load (live and dead) which is transmitted to the girder in question. Statements 1 and 3 are of use in designing trusses.

Note.—The only load applied to a girder between floor beams is its own weight. This is a uniform load and can be considered by itself, according to method previously stated. The following statements do not include this.

1. Shear is constant between any two adjacent floor beams.
2. Moment varies uniformly between any two adjacent floor beams.
3. Moment at any floor beam is the same as it would be if there were no floor beams.
4. If no load is applied in a given panel the moment at any point in that panel is the same as it would be if there were no floor beams.
5. If a load is applied in a given panel of a cantilever girder,

the moment at any point in that panel is *greater* than it would be if the girder had no floor beams.

6. If a load is applied in a given panel of a girder supported at its two ends the moment at any point in that panel is *less* than it would be if the girder had no floor beams.

Problem 50.—Draw shear and moment diagrams for a cantilever girder loaded as shown in Fig. 67.

Problem 51.—Same loading as in problem 50 but no floor beams.

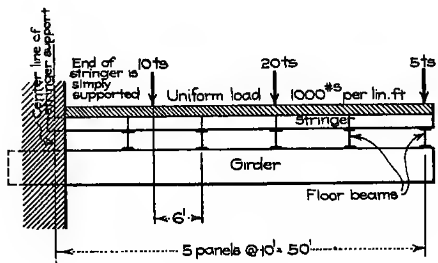

FIG. 67.

Problem 52.—Let the girder shown in Fig. 67 be supported at its two ends. Draw shear and moment diagrams.

Problem 53.—Same as Problem 52 but no floor beams.

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40. A Single Concentrated Moving Load.—For a single concentrated moving load the maximum positive live shear on a simple beam at any section as A, Fig. 68, occurs when the load is just to

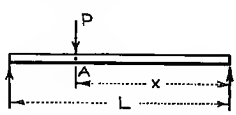

FIG. 68.

the *right* of the section. This may be seen to be true when we consider how the shear varies at the section as a load passes across the beam from the right to the left support. The left reaction and, consequently, the positive shear, is increased as the load P is moved from the right support up to the section, being greatest when the load is just to the right of the section. Now

move the load to the left of A. The shear is equal to the difference between the left reaction and the load P and since a load is always greater than either reaction (the load being equal to the sum of the reactions) the shear, with the load to the left of A, is negative; proving that the positive shear is a maximum with the load just to the right of the section. In practice the load is always placed *at* the section. We might follow through this same line of reasoning for negative shear, moving a load from the left abutment to the section and considering how the shear varies to the right of the section. Thus, in this manner the maximum negative shear will be found to occur when the load is just to the *left* of the section. The value of the maximum positive shear for the load P is $P\frac{x}{L}$, and the maximum negative shear is $P\frac{L-x}{L}$.

The maximum live moment at A occurs with the load at A, for a movement to either side reduces the opposite abutment reaction and, consequently, the moment. The maximum moment is $P\frac{x}{L}(L-x)$.

At any point on a cantilever beam such as A, Fig. 69, the shear is a maximum when the load is anywhere to the right of the

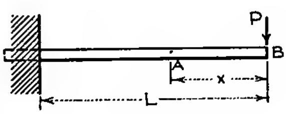

Fig. 69.

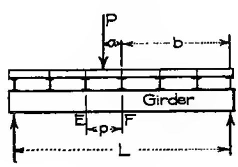

Fig. 70.

point. When the load is on the left the shear is zero. The moment is a maximum at the section when the load is at B and equals $P(x)$. When the load is to the left of A the moment equals zero.

Let us now consider a girder supported at both ends and carrying floor beams. Required the maximum live shear in any panel as EF, Fig. 70. As previously mentioned the load shown is the proportional part of the total load in the panel which is transmitted to the girder in question. The shear is constant in EF for

any loading. Let V denote this shear. Then when the load P is in the panel EF the shear

$$V = (\text{left reaction}) - (\text{load at } E) = P\left(\frac{a+b}{L} - \frac{a}{p}\right).$$

If the load is so placed that $\frac{a+b}{L} = \frac{a}{p}$, then the shear in $EF=0$. This point is called the *neutral* point in the panel. A load to the right of this neutral point causes positive shear and to the left causes negative shear. Every panel has a neutral point which can be found by using the equation

$$\frac{a+b}{L} = \frac{a}{p} \text{ which gives } a = \frac{pb}{L-p}$$

It can be seen from the equation that the position of the neutral point does not depend upon the magnitude of the load but simply upon the length of panel and the position of the panel in the span. The maximum positive shear in panel EF will occur when the load P is at the panel point F, since the shear decreases as the load is moved from that point to the neutral point where it is zero. For the same reason the maximum negative shear will occur when the load is at the panel point E.

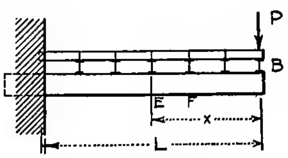

FIG. 71.

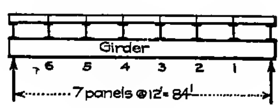

FIG. 72.

We have previously learned that the moment at any point in a panel as EF for a load P in that panel is less than it would be if there were no floor beams, while with the load P outside of EF, the moment is the same as for a simple beam. At the floor beams the moment is the same as if there were no floor beams. In designing structures we are only interested in maximum moment, consequently, it is sufficient to compute the moments only at the floor beams and to do it just as if there were no floor beams. Fig. 71 represents a cantilever girder supporting floor beams. Maximum shear in EF occurs when the load is anywhere to the right of F and equals P. Maximum moment at any panel point as E occurs with P at B and equals $P(x)$.

Problem 54.—A 5000 lb. concentrated load passes back and forth over the girder shown in Fig. 72. Find the maximum positive and maximum negative live shear in panel 3–4, and the maximum live moment at panel point 3. Also find neutral point in panel 3–4.

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41. Moving Uniform Load.—For a moving uniform load the maximum positive live shear on a simple beam at any section as A, Fig. 73, occurs when the right hand section of the beam is loaded up to the point considered.

This is seen to be true when we consider that adding a load to the right of A increases the left reaction and, therefore, the positive shear, while adding a load to the left of A, increases the left reaction by an amount less thân the load which is added and, hence decreases the positive shear. The maximum positive shear at A in Fig. 74 for a uniform load of w pounds per foot $= \frac{1}{2} w \frac{x^2}{L}$.

From similar reasoning to the above, the maximum negative shear at any section as A, Fig. 73, is found by loading to the left of the point. Maximum negative shear at A, Fig. 75, for a uniform load of w pounds per foot $= \frac{1}{2} w \frac{(L-x)^2}{L}$ (considering the right hand reaction).

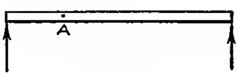

FIG. 73.

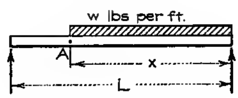

FIG. 74.

The maximum moment at any section as A occurs when the beam is *fully loaded*, for the addition of a load anywhere on the beam will add a positive moment at the section. For a load of w pounds per foot, the

$$\text{maximum } M = \frac{wL}{2}(L-x) - \frac{w(L-x)^2}{2} =$$

$$\frac{w}{2}(L-x)(L-L+x) = \frac{w}{2}(x)(L-x).$$

If the section is at the center of the beam, the

$$\text{maximum } M = \frac{1}{8} wL^2.$$

The student is advised to memorize this equation.

The above formulas for maximum moment give results in foot pounds since w represents the load in pounds per foot and L the span of the beam in feet. To get inch pounds multiply by 12 or insert for w in the formulas the load in pounds per inch and for L the span of the beam in inches.

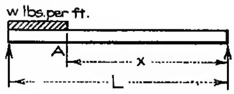

Fig. 75.

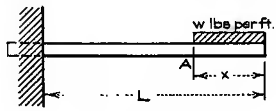

Fig. 76.

At any point on a cantilever beam such as A, Fig. 76, the maximum shear occurs for either a full load over the entire length, or for full load on the portion of the beam between the section and the free end, and equals wx. The moment is alway negative and the maximum moment occurs for the same loading giving maximum shear; namely,

$$\text{maximum } M = \frac{wx^2}{2}.$$

Let us now consider the case of a uniform load of w pounds per foot on a girder supported at its two ends and carrying floor

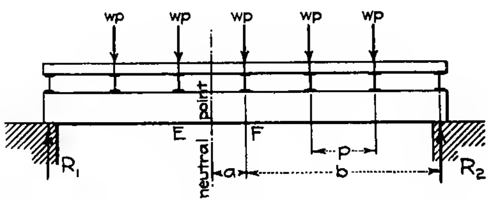

Fig. 77.

beams. If the girder is fully loaded the load on each floor beam is wp, except on the end floor beams, which carry $1/2\ wp$. This last load is really supported directly on the abutments, and should always be neglected in determining shear and moment. R_1, Fig. 77, then equals $2\ 1/2\ wp$ and R_2 equals $2\ 1/2\ wp$. The maximum positive shear in any panel, such as EF, occurs when the load extends from the right up to the neutral point in the panel (Fig. 78). Thus,

$$\text{maximum } V = \frac{w(a+b)^2}{2L} - \frac{wa^2}{2p}.$$

In practice, the assumption is generally made, that for maximum positive shear in a panel, all panel points up to and including the one at the right of the panel are fully loaded, and the ones to the left without any load. It is not possible to get this loading, but it is convenient and a little on the safe side. We know that in order for panel point F, Fig. 78, to have a full load, the load must extend to the panel point E and then E would have half a panel load. A load at E would reduce the positive shear in EF so

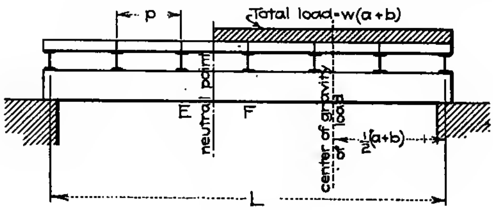

FIG. 78.

by omitting this we are on the safe side; that is, we are providing for a little greater positive shear than actually exists. For this loading the shear in EF is

$$\frac{(1+2+3)}{6}\ (pw).$$

The maximum negative shear is likewise

$$\frac{(1+2)}{6}\ (pw).$$

The moments at the floor beams are the same as they would be if there were no floor beams. Maximum moment occurs as before for full loading and is positive at every point. The maximum moment at a floor beam distant x from the right abutment is (as in a simple beam)

$$\frac{wL}{2}\ (L-x)-\frac{w\,(L-x)^2}{2}=\frac{w}{2}\ (x)(L-x).$$

Fig. 79 represents a cantilever girder supporting floor beams. Maximum shear in EF occurs when BE is loaded and equals $w(b+\frac{1}{2}\ p)$. Maximum moment at E occurs for either full loading or for full load on BE, and equals (in this particular figure),

$$p(1+2+3)\ wp+4p\ (\frac{1}{2}\ wp)=p^2w\ (1+2+3+2)=8p^2w.$$

Problem 55.—Compute maximum positive shear at sections *a*, *b*, and *c* of beam shown in Fig. 80 for a uniform live load of 2000 lb. per lin. ft. and a uniform dead load of 200 lb. per lin. ft. (Remember dead load covers entire span and the maximum positive shear is required for the live and dead loads together. Place the live load so as to get the greatest positive shear.)

Problem 56.—Compute maximum moments at sections *a*, *b*, and *c* of beam shown in Fig. 80 for a uniform live load of 2000 lb. per lin. ft. and a uniform dead load of 300 lb. per lin. ft.

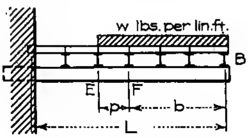

Fig. 79.

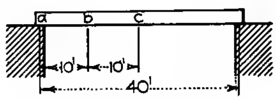

Fig. 80.

Problem 57.—A truss is 60 ft. long and has 6 equal panels. Compute shear in each panel and moment at each floor beam caused by a uniform dead load of 600 lb. per lin. ft. applied to the girder at the floor beams.

Problem 58.—A truss is 60 ft. long with 6 equal panels. Compute maximum positive shear in each panel by the approximate method, and the maximum moment at each floor beam caused by a uniform live load of 2000 lb. per ft.

Problem 59.—Combine results of problems 57 and 58 so as to obtain the maximum positive shear in each panel and the maximum moment at each floor beam due to both the dead and live uniform loads.

CHAPTER VI

INFLUENCE LINES

42. Method of Drawing Influence Lines.—As a load moves over a girder or truss, the shear and moment at a given point will vary. Suppose the load considered is a unit load and let us plot ordinates representing the function we are studying at each successive position of the load. Plot positive values above a given line (which we shall call the *base-line*) and negative values below this line. The line joining the ends of the ordinates is called an *influence line* and shows how the moment or shear (or whatever function we are studying) varies at some definite point as the load of unity passes over the structure. (Figs. 81, 82, 83, and 84).

43. Use of Influence Lines.—The influence line shows three things:

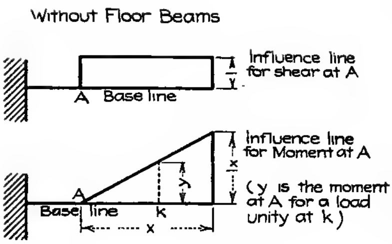

Fig. 81.

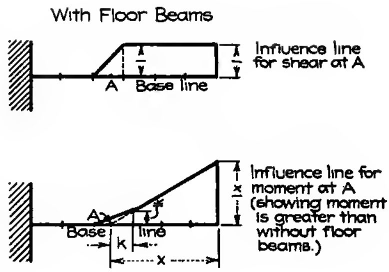

Fig. 82.

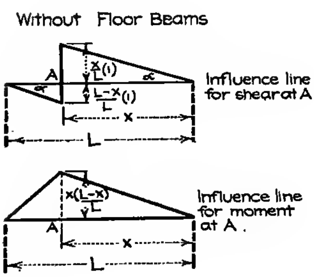

Fig. 83.

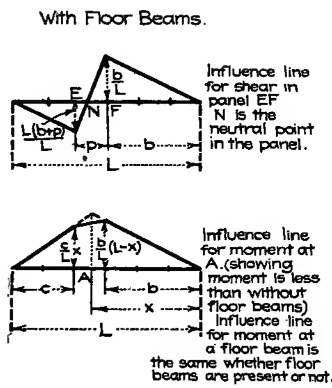

Fig 84.

1. The effect on the function we are studying for a single load at any point on the structure.

2. Where a single load must be placed in order to produce the maximum or minimum effect.

3. With a uniform live load, the part (or parts) of the structure which must be loaded in order to produce the maximum positive or the maximum negative effect.

Consider a girder to have six equal panels and suppose that the influence line for shear is desired in the second panel from the left end. To draw this line it is necessary to consider a load of unity to pass over the structure from right to left. There will be a uniform increase in the left reaction, and hence in the shear in panel BC (Fig. 85), as the unit load passes from the right end

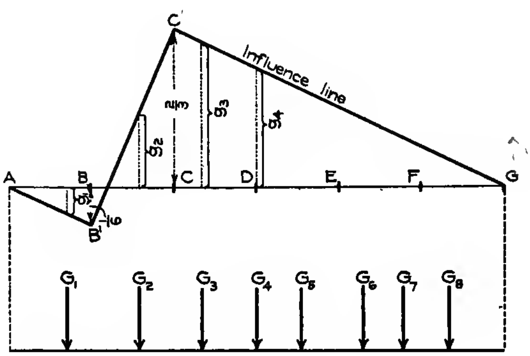

FIG. 85.

of the structure to the panel point C. Imagine ordinates to be plotted at each successive position of the load as the load moves between these points, then the line joining the ends of these ordinates will be straight. At the point C the value of the ordinate to the influence line is 4/6 multiplied by the load $(1) = \frac{2}{3}$. If the load unity is now considered to move beyond C to the left, the left reaction keeps on increasing, but a part of the unit load now goes to panel point B. The load this panel point receives continually increases with the result that the positive shear in panel BC decreases. This decrease takes place uniformly (shown by the straight line $C'B'$) until panel point B is reached

where the ordinate to the influence line has a negative value of $\frac{1}{6}(1)=\frac{1}{6}$. Now as the unit load moves from B to the left end, the shear in panel BC varies directly as the right reaction and hence there is a uniform decrease of negative shear until the value of the shear becomes zero at the left support. The influence line for shear in panel BC is now completely determined.

Imagine a system of loads to be standing on the structure in the position shown and at fixed distances apart, similar to locomotive wheel loads. A load of unity at the point of application of G_1 produces an effect measured by the ordinate g_1. A load of magnitude G_1 must then produce an effect which is equal to g_1G_1. In like manner G_2 produces an effect which is equal to g_2G_2, etc. All these loads then in this position produce an effect equal to $g_1G_1+g_2G_2+g_3G_3+$etc., if due regard is paid to signs. The total effect might then be better expressed by saying that it is equal to the *algebraic* sum of the effects of the separate loads, the effect of any given load being equal to the ordinate at its point of application multiplied by its magnitude.

It can also be easily seen from the above that if a uniform load extended over the structure, the entire effect could be measured by the algebraic sum of the areas included between the influence line and the base-line multiplied by the weight of the uniform load in pounds per linear foot. If the load extended only as far as C the effect would be the area $CC'G$ multiplied by the weight in pounds per linear foot.

Influence lines are not generally used for determining values of functions for simple beams, girders, or trusses, because the algebraic methods are more simple, but the use of influence lines leads to a better understanding of the effect of moving loads and in many complicated structures the influence line affords the simplest and best solution of a problem. It is freely used in methods of analysis; that is, finding the position of loads to give maximum shear or moment or whatever the function may be which is under consideration.

To illustrate in a general way, suppose the loads shown in Fig. 85 are allowed to roll back and forth along the structure; the loads keeping their fixed distances apart. Let the loads come on the structure from the right. The function will be increased until the load G_1 reaches the point C. After G_1 passes C the function will decrease considering only G_1, but it may

increase because of the other loads moving up to the left. It should be observed since GC' and $C'B'$ are straight lines, that no matter whether the function is *increasing* or *decreasing* after G_1 passes C it will keep on doing so until B is reached; providing, however, that load G_2 in the meantime does not pass point C and that no additional load comes on the structure from the right. If a load passes C or comes on the structure from the right, conditions are altogether changed.

From the above discussion, it should be clear, that to obtain maximum values of the function, whether positive or negative, it is necessary to stop moving the loads when *any* load reaches an angle in the influence line and compute the value of the function for each of these positions. The greatest of all these maximum values will be the true maximum value or what is called the *maximum maximorum.*

There is one exception to the above statement. If we are trying to obtain the maximum $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$ value of a function and a load reaches an angle point in the $\left\{\begin{matrix}\text{negative}\\ \text{positive}\end{matrix}\right\}$ part of the influence line, then it is not necessary to obtain a value of the function with the given load at that point if the angle at that point included within the influence diagram is less than 180°. For instance, if we are trying to obtain maximum positive shear, and a load should arrive at point B, Fig. 85, it is not necessary to determine the value of the function for this position of the loads, because a load at B will tend to *increase* the function if moved still farther to the left.

Problem 60.—A girder 55 ft. long has 5 equal panels. Draw the influence line for shear in the second panel from the left support.

Problem 61.—By means of influence lines determine where a uniform load of g lb. per ft. must be placed to obtain maximum positive and negative shear and maximum moment at section A, Fig. 73, and then find the values of these maximum functions.

Problem 62.—A structure is 40 ft. long and consider the influence line for a given function as shown in Fig. 86. The loads shown pass back and forth over the structure. Determine the position of the loads which produces the greatest positive effect and also determine the corresponding value of the function.

Note to Problem.—The student should lay off a horizontal line to represent the length of the structure and then erect the ordinates at the given points to

some definite and sufficiently accurate scale. Plot the loads on the edge of a separate strip of paper; the distance apart of the loads to be to the same scale as the length of the structure. Beginning with the strip of paper entirely to the right of the span move the strip of paper to the left until the first load reaches an angle point and determine the corresponding value of the function by scaling the ordinates at the given loads and multiplying each ordinate by the corresponding load and then finding the algebraic sum of these products. Now move again to the left until some load reaches an angle point. Scale the ordinates and determine the corresponding value of

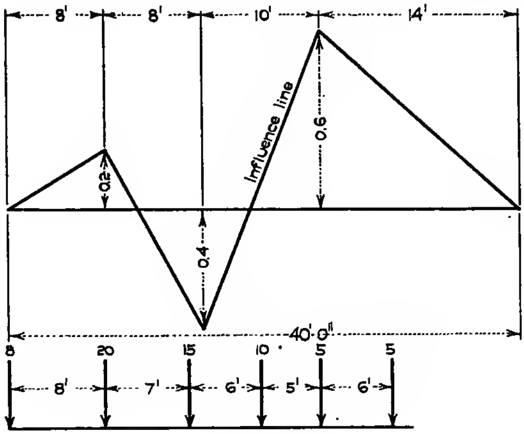

FIG. 86.

the function as before. This process should be continued until the loads have all passed off the structure to the left. Examine the different values obtained and pick out the *maximum maximorum*.

Problem 63.—With the loads given in Fig. 86, determine the maximum positive shear and maximum moment at A, Fig. 83. Suppose the span is 60 ft. and that A is 16 ft. distant from the left support. Use the influence line method of Problem 62.

Problem 64.—With the loads given in Fig. 86 determine the maximum positive shear in the panel EF, Fig. 84. Let the span be 60 ft. and the panels equal. Use the influence line method of Problem 62.

After working out Problems 63 and 64 the student should observe the following facts:

a. For the maximum $\left\{\begin{matrix}\text{positive}\\\text{negative}\end{matrix}\right\}$ shear at any section, without floor beams, the loads must be as completely as possible to the $\left\{\begin{matrix}\text{right}\\\text{left}\end{matrix}\right\}$ of the section, one load must lie at the section and the heaviest loads must lie as near the section as practicable.

b. In a structure with floor beams, the maximum $\left\{\begin{array}{c}\text{positive}\\ \text{negative}\end{array}\right\}$ shear in any panel as EF, Fig. 70, will occur when there are few loads to the $\left\{\begin{array}{c}\text{left}\\ \text{right}\end{array}\right\}$ of $\left\{\begin{array}{c}E\\ F\end{array}\right\}$.

c. The moment at a panel point where there are floor beams (or at *any* point when there are no floor beams), is a maximum when some load lies at that point, and when the heaviest loads are nearest the point.

ASSIGNMENT 8

CHAPTER VII

CONCENTRATED LOAD SYSTEMS—ALGEBRAIC TREATMENT

44. Moment Table.—The position of a system of concentrated loads to give either maximum live shear or maximum live moment at any point of a girder or truss, as well as the numerical values of these maximum shears and maximum moments, can be conveniently determined by means of a moment table. After once a moment table is constructed for a given system of loads, it can be used for any number of girders or trusses with different spans and panel lengths.

In this article we shall simply describe the construction of such a table, leaving its use to be explained later.

Fig. 87 shows a moment table for a system of loads consisting of two locomotives followed by a uniform train load. An examination of the table will show that it gives the axle loads with their numbers and spacing; the distances and sum of loads from any given load to load No. 1; and the moment of all preceding loads with reference to the axis of each wheel. Since the majority of railroad bridges are single-track with two girders or trusses, the loads shown are those upon one rail and, consequently, the shears and moments obtained will be the shears and moments for just one girder or truss.

The only part of the table requiring explanation is the line of figures giving the moment at each load due to all the preceding loads. The moment at load No. 5, for example,

$$
\begin{aligned}
&= (\text{load } 1)\ (23) + (\text{load } 2)\ (15) + (\text{load } 3)(10) + (\text{load } 4)(5) \\
&= (12)\ (23) + (24)\ (15) + (24)\ (10) + (24)\ (5) \\
&= 996{,}000 \text{ ft. lb.}
\end{aligned}
$$

To work out each one of these moments in this way is quite laborious and a more simple method may be used. In this simple method each moment is derived from the moment preceding by

computing the moment increment. The moment increment from load 4 to load 5 equals

$$(84)(5) = 420$$

and 420 added to 576 gives 996. The rule for deriving the

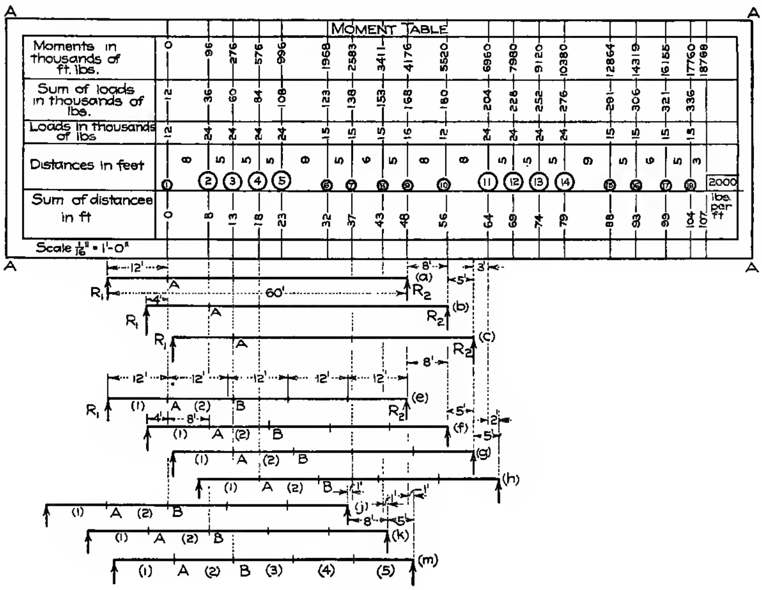

FIG. 87.

moment at any given load from the moment at the load preceding, may be expressed as follows: To obtain the moment at any given load multiply the sum of all the preceding loads by the

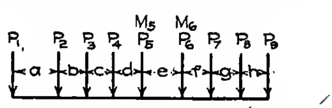

FIG. 88.

distance between the given load and the next load preceding and add the moment at the preceding load.

To show that the increment method gives correct results, let us consider the load shown in Fig. 88 and let the moment at load P_6 be required.

$$M_5 = P_1(a+b+c+d) + P_2(b+c+d) + P_3(c+d) + P_4(d)$$
$$M_6 = P_1(a+b+c+d+e) + P_2(b+c+d+e) + P_3(c+d+e) + P_4 (d+e) + P_5 e.$$
$$M_6 = M_5 + P_1 e + P_2 e + P_3 e + P_4 e + P_5 e$$
$$M_6 = M_5 + (P_1 + P_2 + P_3 + P_4 + P_5)e$$

which proves the increment moment rule stated above.

Problem 65.—Construct a moment table for the loads shown in Fig. 89.

---

45. Maximum Shear With and Without Floor Beams.—Before we can find the value of maximum shear at a given section, it is first necessary to find just how the loads must be placed in order to give this maximum shear. Consider now the position of loads to give maximum positive shear at any section of a simple beam. Denote the section by the letter A, Fig. 87a.

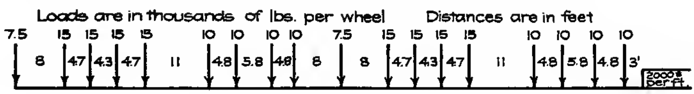

Fig. 89.

Let the loading come on from the right and move up until the first load of the moment table is at the section we are considering. Then the question arises, shall we gain or lose by moving still farther to the left? Fig. 87a shows the first load at section A. If load 1 goes to the left of A, the left hand reaction increases, but at the same time load 1 must be subtracted from the said reaction. If we move up at all, we must move up until load 2 comes to the section (see influence line). Fig. 87b shows load 2 at A. Has this shear been increased or decreased by moving up the second load? If any load denoted by the letter P be moved a distance d to the left, then the left reaction is increased by the amount $P\frac{d}{L}$, L being the span length. Hence, by moving load 2 up to the section, loads 1 to 9 inclusive move a distance of 8 ft. to the left and R_1 is increased by

$$(168)\left(\frac{8}{60}\right).$$

What amount of shear have we lost in so doing? It should be

clear that we have lost only the weight of load $1=12$. Thus, if $(168)\left(\frac{8}{60}\right)$ is greater than 12, the shear has been increased by moving to the left.

$(168)\left(\frac{8}{60}\right)=22.4$ and thus >12.

($>$ means *greater than* and $<$ means *less than.*)

Since the shear is greater with the second load at A than with the first load, let us see if the shear will be still further increased by moving up load 3. Loads 1 to 10 inclusive move a distance of 5 ft. to the left. (Fig. 87c). This time load 1 goes off the span. The question is, did we gain or lose by moving up the loads? Increase in left reaction, not considering the effect of load 1, is

$$(180-12)\left(\frac{5}{60}\right)=14.$$

The increase in shear due to load 1 going off the span is

$$(12)\left(\frac{4}{60}\right)=0.8$$

and equals the value of the right reaction due to load 1 when load 2 is at A; in other words, the right reaction due to load 1 with load 2 at A measures the shear gained.

The shear is decreased by the load 2 going to the left of the section; that is, by the amount 24.

The above expressed as one equation is

$$14+0.8 \gtrless 24.$$

(This equation put into words is as follows: Is $14+0.8$ greater or less than 24?)

$$14.8<24.$$

Thus, the shear is decreased by moving up load 3 to A and the shear is a maximum with load 2 at the section.

It is seen by inspection that in this case it is unnecessary to try moving up either load 4 or load 5 and that under the first set of drivers load 2 is the only load giving maximum shear. A continued movement of the loads will eventually cause a minimum shear and later on a maximum will result with some load under the second set of drivers.

When a load comes on the structure from the right, in the process of moving up the loads, the increase of shear due to this

load is usually so small that it can be omitted in all cases except when the inequality becomes very nearly an equality.

The above might be called the *gain or lose* method of determining the position of loads to give the maximum shear.

In this article, we have, for clearness, shown the beam as moved to suit the several conditions, but in practice the moment table is cut out along the lines between points $AAAA$ and the diagram is shifted back and forth along a line drawn to represent the beam. It should be clear that the line representing the beam must be drawn to the same scale as the moment table.

We already know from the study of influence lines that it is not necessary to try for maximum shear except with the heaviest loads at the section. It is only necessary, in the above case, to try loads 2, 3, 4, and 5. A maximum will surely be found under one of these four loads, generally under the second or third. A maximum will also come under one of the loads, 11, 12, 13, or 14. This maximum, in some instances, will be greater than the first maximum found and it is always necessary for the student to consider this.

Another method of determining the position of loads algebraically to give maximum shear on simple beams, due to a system of concentrated loads, is given in Art. 51. The method is explained in connection with the graphical method of determining values of maximum shear.

Problem 66.—Find the position of the live loads shown in Fig. 87 that will cause maximum positive shear at sections a, b, and c of the beam shown in Fig. 80. Consider only the heavy loads of the first locomotive.

We have found that for the beam in question, the shear is a maximum with load 2 at section A, Fig. 87b. It will now be shown how the table may be used to determine the value of this maximum shear. We make use of the formula $\Sigma M=0$.

The moment at load 10 of all the preceding loads $=5520$ and equals the moment at the right support.

The left reaction is 5520/60. The load 1 is to the left of A and, hence, the maximum shear at the section is

$$\frac{5520}{60}-12=80{,}000 \text{ lb.} \quad \textit{Answer.}$$

Problem 67.—Find the value of the maximum live shear at points *a*, *b*, and *c*, Fig. 80, for the positions determined in Problem 66.

The position of a system of concentrated loads to give maximum shear in any panel of a structure containing floor beams can be obtained by the same method as explained above. In this case, however, it is important to remember that the load in a panel is distributed proportionately to the adjacent floor beams, the proportion each floor beam receives depending upon the position of the load in the panel.

It is stated in Art. 32 that the stringers in the end panels of a bridge either rest directly upon the abutments or upon end floor beams. In either case an end panel load is fully carried by the support beneath, thus causing no shear or moment on the girder or truss. It is for this reason that the same results will be obtained in computing shear and moment no matter what type of end construction is considered. An example will serve to bring out what is meant.

Consider load No. 1 in panel (1) of the girder or truss shown in Fig. 87f, and assume no end floor beams. Then the shear due to this load is

$$\left(\frac{4}{12}\right)(12)\left(\frac{4}{5}\right)=3.2$$

Assuming now that the stringers rest on floor beams at the end, the shear is

$$\left(\frac{56}{60}\right)(12)-\left(\frac{8}{12}\right)(12)=3.2$$

as before. Whether end floor beams are assumed or not should thus be governed by convenience.

Take the girder or truss shown in Fig. 87e. There are 5 panels of 12 ft. each making a total span of 60 ft. Let the maximum shear be required in panel (1). The left reaction and, consequently, the shear in (1) will surely be increased by moving up the loads from the right until load 1 is just to the right of the panel. Fig. 87e shows the loads in this position.

The question is, as before, shall we gain or lose by moving up the second load? Fig. 87f shows the loads in the moved posi-

tion; that is, with load 2 at the panel point A. The expression for what is gained or lost assuming end floor beams is as follows:

$$(168)\left(\frac{6}{60}\right) \gtrless \left(\frac{8}{12}\right)(12)$$

The quantity $\left(\frac{8}{12}\right)(12)$ is the amount of load 1 which goes to the left of panel (1); that is, to the support R_1.

$$22.4 > 8.$$

Consequently, the maximum shear in panel (1) is increased by moving up load 2 to A. Will it be still further increased by moving up load 3 (Fig. 87g)? Load 1 goes off the span. Load 10 comes upon the structure from the right and, assuming end floor beams,

$$(180-12)\left(\frac{5}{60}\right) + \left(\frac{8}{12}\right)(12) \gtrless \left(\frac{56}{60}\right)(12) + \left(\frac{5}{12}\right)(24).$$

$\left(\frac{56}{60}\right)(12)$ is the amount of the left reaction which is lost by load 1 going off the span. $\left(\frac{8}{12}\right)(12)$ is the amount by which load 1 decreased the shear before load 3 was moved up to the section and for that reason the shear is increased by that amount when moving up the loads. $\left(\frac{5}{12}\right)(24)$ is the decrease in the shear due to the load 2 going to the left of A.

The total amount of shear that is lost by load 1 going off the span is

$$\left(\frac{56}{60}\right)(12) - \left(\frac{8}{12}\right)(12) = 3.2$$

This may be computed in a more simple manner by considering no end floor beams as far as load 1 is concerned. Before load 1 is moved off the span, the part of the load which goes to panel point A increases the left reaction by $\left(\frac{4}{12}\right)(12)\left(\frac{4}{5}\right) = 3.2$; considering the panel length a unit distance. $\left(\frac{4}{12}\right)(12)\left(\frac{4}{5}\right)$ is thus the total

amount we lose by load 1 going off the span. The above inequality may then be better expressed as follows:

$$(180-12)\left(\frac{5}{60}\right) \gtrless \left(\frac{4}{12}\right)(12)\left(\frac{4}{5}\right)+\left(\frac{5}{12}\right)(24)$$

$$14 > 13.2$$

The left side of the inequality is again greater than the right side and the shear is increased by moving up load 3 to A. Now move up load 4 (Fig. 87h) and assuming end floor beams,

$$(180-12)\left(\frac{5}{60}\right)+(24)\left(\frac{2}{60}\right) \gtrless \left(\frac{10-5}{12}\right)(24)+\left(\frac{5}{12}\right)(24)$$

$$14.8 < 20.$$

Thus, the shear is decreased by moving up load 4 and the maximum shear in panel (1) occurs with load 3 at A.

The value of the maximum shear is

$$\frac{(5520)+(180)(5)-(12)(61)}{60}-\left(\frac{5}{12}\right)(24)=84{,}800 \text{ lb.}$$

Suppose the maximum shear in panel (2) is required. Consider load 1 at B, Fig. 87j. Move up load 2 to B, Fig. 87k.

$$(123)\left(\frac{8}{60}\right)+(15)\left(\frac{7}{60}\right)+(15)\left(\frac{1}{60}\right) \gtrless \left(\frac{8}{12}\right)(12)$$

It is evident that the shear has been increased. Move up load 3, Fig. 87m.

$$(153)\left(\frac{5}{65}\right)+(15)\left(\frac{1}{60}\right) \gtrless \left(\frac{4}{12}\right)(12)+(24)\left(\frac{5}{12}\right)$$

$$13.0 < 14.0$$

Consequently, the shear is decreased by moving up load 3 and the maximum shear in panel (2) occurs with load 2 at B.

Maximum shear is also likely to occur under the second set of drivers, as explained for girders without floor beams (which see).

The value of the maximum shear in panel (2) is

$$\frac{3411+(153)(1)}{60}-\left(\frac{8}{12}\right)(12)=51{,}400 \text{ lb.}$$

It should be clear to the student that the maximum positive shear in panel (1) is the same amount as the maximum negative shear in panel (5). (Fig. 87m) and likewise, the maximum positive shear in (2) is the same as the maximum negative shear in (4).

Thus, it is evident that the maximum positive shears are needed only in panels (1), (2), and (3), and the maximum shears in all the panels will be determined.

There are no established formulas for finding the position of loads to give maximum shear by the *gain or lose* method. Conditions may vary greatly. Different systems of loads (unless standard) have different spacing and the panel lengths are very seldom the same in any two bridges considered. It is thus evident that formulas could not easily be prepared to cover all cases even if one were inclined to use them. The only rule to be followed in this method is to move up one load at a time and find out if the shear has been increased or decreased by so doing. Form the equations intelligently, being sure that the expression to the left of the inequality sign represents *gain* in shear and that to the right of this sign represents *loss*. When the point has been reached, where the shear is decreased instead of being increased, then the position of the loads to give maximum shear is at once determined.

Another method of determining the position of loads algebraically to give maximum shear in any panel of a structure containing floor beams, due to a system of concentrated loads, is given in Art. 53. The method is explained in connection with the graphical method of determining values of maximum shear.

Problem 68.—Consider the 40 ft. girder shown in Fig. 80 to have 4 equal panels. Loading as shown in Fig. 87. (a) Determine the position of loads to give maximum positive live shear in the first and second panels from the left support. Consider only the heavy loads of the first locomotive. (b) Compute the value of the maximum live shear in each panel for the position of loads determined in part (a).

ASSIGNMENT 9

CHAPTER VII—*Continued*

46. Maximum Moment With and Without Floor Beams.— Before we can determine maximum live moment at any point of a girder or truss for a system of concentrated loads it is necessary to find the position of the loads to give this moment.

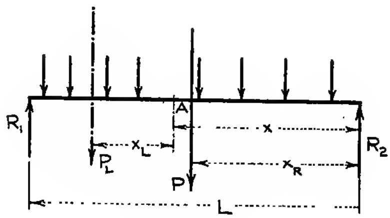

FIG. 90.

Let us consider the determination of maximum moment at a section of a simple beam, such as A, Fig. 90.

Let P_L = resultant of all loads to the left of A.
x_L = its distance from the section.
P = total load on span.
x_R = its distance from right support.
x = distance of section from right support.

Then the moment at A is:

$$M = P\frac{x_R}{L}(L - x) - P_L x_L.$$

Let the system of loads be moved a small distance $\triangle$ to the

left, the distance being so small that the distribution of the loads will not be changed. Then the new moment is:

$$M' = P\frac{x_R+\triangle}{L}(L-x) - P_L(x_L+\triangle)$$
$$= \left[P\frac{x_R}{L}(L-x) - P_Lx_L\right] + P\frac{\triangle}{L}(L-x) - P_L\triangle$$

The moment has increased by so doing provided

$$P\frac{\triangle}{L}(L-x) > P_L\triangle$$
$$\text{or } P\left(\frac{L-x}{L}\right) > P_L$$

or $\frac{P}{L} > \frac{P_L}{L-x}$ [dividing each side of the inequality by $(L-x)$.]

In other words, the moment at a given section will keep increasing by moving the loads to the left until the sign of inequality is changed. That is, the maximum moment is obtained when with a load to the right of the section

$$\frac{P}{L} > \frac{P_L}{L-x}$$

and with the same load moved to the left of the section

$$\frac{P}{L} < \frac{P_L}{L-x}.$$

During this slight movement $\frac{P}{L}$ passes the value $\frac{P_L}{L-x}$.

Thus, for maximum moment

$$\frac{P}{L} = \frac{P_L}{L-x}$$

It follows from this that the moment will be increased by moving the loads to the left provided the average load per foot on the whole span is greater than the average load on the left of the section. Thus, the moment at any section, as A, will occur when some load lies at that point, and that load must be such that when it lies just to the right of the section, the average load on the whole span will be greater than the average on the left, while if it lies to the left of the section, the average load on the left will be the greater.

Let us find the position of the loads to give maximum moment at section A, Fig. 87a. Starting with load 1 at the section, will the moment be increased by moving up load 2? (Fig. 87b). With load 1 to the right of A the average load on the whole span is without question greater than the average on the left of the section. With load 1 just to the left of A the average load on the left is $\frac{12}{12}$ and the average load on the whole span is $\frac{168}{60}$.

$$\frac{168}{60} > \frac{12}{12}$$

Not a maximum. Move up.

It is not necessary to consider load 2 just to the right of the section since no load has passed to the left of A to increase the average load on the left and we have just seen that the average load on the left must become greater than the average on the whole span in order to procure a maximum. The student should realize, however, that if a start is made in applying the criterion with any other load than load 1 at A, the first load considered must be tried both to the right and left of the section.

With wheel 2 just to the left of A, Fig. 87b,

$$\frac{180}{60} = \frac{36}{12}.$$

This means that the moment which has been increasing up to this time by moving the loads to the left, will now remain the same until some load either comes on the span, passes the section, or goes off the span. If a load comes on the span, the moment is increased and the loads should be kept moving to the left. If a load should go off the span before a load reaches the section, then the average load on the whole span is still greater than the average load on the left, and the moment will keep increasing until some load reaches the section. Thus it follows from the above, that when the average load on the whole span is equal to the average load on the left of the section, the resulting moment is not necessarily a maximum. It is a maximum only when no load comes on or goes off the span in the process of moving up the next load to the section. In such a case the same maximum moment is obtained for the two loads in succession. In the problem at hand it is clear we have not this condition and wheel 2 at A, consequently, will not give a maximum moment.

With load 3 at the left of A, Fig. 87c,

$$\frac{180-12}{60} \gtrless \frac{60-12}{12}$$

$$2.8<4.$$

Maximum moment occurs then with load 3 at A. It is seen by inspection that in this case it is unnecessary to try loads 4 and 5 at the section.

By the use of the diagram the value of the maximum moment is

$$\frac{5520+(180)\ (5)-(12)\ (61)}{60}\ (12)-(24)\ (5)=1{,}017{,}600 \text{ ft. lb.}$$

A maximum moment will also occur under the second set of drivers and it sometimes happens that more than one maximum will occur under *each* set. Generally, it is easy to see what position of the loads will give the *maximum maximorum,* but sometimes it is necessary to compute several maximum moments. If there is any doubt regarding the *maximum maximorum* compute all the possible cases.

Consider now a structure having floor beams. As previously explained, the moment between floor beams is always less than if there were no floor beams. Hence, it is only necessary to compute the maximum moments at the floor beams and to do it as if there were no floor beams.

Problem 69.—By means of influence lines show that the maximum live moment at the first panel point A, Fig 87e, equals the maximum live shear in panel (1) multiplied by the panel length.

Problem 70.—Consider the 40 ft. girder shown in Fig. 80 to have 4 equal panels. Loading as shown in Fig. 87e. (a) Determine the position of loads to give maximum moment at panel points b and c. (b) Compute the value of the maximum moment in each case for the position of loads determined in part (a). *Note.*—In this problem consider the heavy loads of both locomotives.

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47. Absolute Maximum Moment.—A stringer is a simple beam and the moment and shear is computed as for *any* simple beam. In the design of simple beams, it is necessary to determine the value of the maximum moment. For uniform loading, it occurs at the center of the span with the beam entirely loaded. If two stringers are used in a panel length of the bridge, and, if they are

equally spaced about the center line, each stringer will receive one-half the total live panel load. With concentrated live load systems there is no reason why the maximum moment should occur at the center and, in fact, it will generally not occur there. Hence, it is necessary first of all to determine the *section* of greatest moment.

In the preceding article the method was explained for finding maximum moment at given sections. Such a method is very valuable when computing moments on girders or trusses having floor beams. In simple beams, however, such as stringers and beam bridges, it is always important to find the value of the moment at the section where the moment is the *greatest.* We shall call this greatest moment the *absolute maximum moment.* It is the greatest moment that can possibly come upon a simple beam for a given system of concentrated loads.

Imagine a series of concentrated loads to move over a beam. The bending moment at any wheel load will vary as the loads move over the structure. We shall now find the maximum

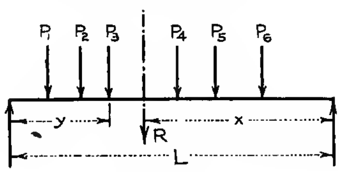

FIG. 91.

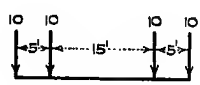

FIG. 92.

moment at one of the moving loads, after determining just how the loads shall be placed on the span so as to cause said maximum moment at the load chosen. We have only to do this for each of the heavy loads and the greatest of these will be the absolute maximum sought.

Suppose the maximum moment is required at the load P_3, Fig. 91, as the load system passes over the span. Let R equal the resultant of all of the loads on the span when P_3 is somewhere near the center of the beam. The moment at P_3 is:

$$M_3 = R\frac{xy}{L} - (\text{moments of loads } P_1 \text{ and } P_2).$$

In order for M_3 to be a maximum we must have xy a maximum; that is, x must equal y. In other words, the center of the beam must be half way between P_3 and R.

Thus, the method of determining the maximum moment under any one of the concentrated loads is to place the loads so that the load in question is near the center of the beam. (It is more convenient to move a line representing the length of the beam than it is to move the loads). Find the line of action of the resultant of the loads which are on the span when the loads are placed as directed above. Now place the beam so that its center will come midway between R and the load in question. If now one of the loads which we assumed would be on the span is found *not* to be on the span, we must leave that load out and again compute R and its line of action. Now compute the maximum moment at the load considered. The maximum moment should next be found at each of the heavy loads in the same manner as above. The greatest moment will be the absolute maximum.

Problem 71.—Find the absolute maximum moment on a simple beam of span 40 ft. for the loads shown in Fig. 92.

Problem 72.—The absolute maximum moment is required on a beam 24 ft. in length for the system of concentrated loads shown in Fig. 87.

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48. Maximum Floor Beam Reaction.—The load upon floor beams comes from the adjacent stringers. The stringers are generally equally spaced about the center line between girders

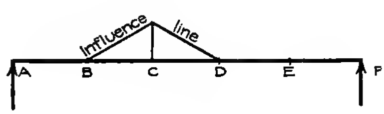

Fig. 93.

or trusses. Floor beams are thus simple beams with concentrated loads at definite points. In a single-track railroad bridge the stringers are equally spaced about the center line, one or two being placed under each rail. If there are two stringers in each panel, one under each rail, each stringer receives one-half the trainload.

Floor beams are computed as any simple beam would be. It is important, however, in designing floor beams to determine the maximum load which can come from the adjacent stringers. Suppose the maximum floor beam load is required at C, Fig. 93.

The influence line for the load coming upon C has the shape as shown. This is the same shape as the influence line for the moment at C on a beam of length BD. The condition of loading for maximum floor beam reaction at C is thus the same loading that will give maximum moment at C on a beam of span BD. The method of finding the position of loads to give maximum moment has already been explained.

Problem 73.—A single-track railroad bridge has one stringer under each rail. The loads in thousands of lb. per wheel are shown in Fig. 87. There are 4 equal panels in a span of 40 ft. Determine the maximum floor beam load under each line of stringers for a floor beam other than at the end of bridge.

Problem 74.—The railroad bridge described in Problem 73 is 14 ft. between girders. The stringers are spaced 6 ft. center to center. The dead loads are as follows:

track – 400 lb. per ft.
one stringer – 75 lb. per ft.
one floor beam – 100 lb. per ft.

Determine: (a) – Maximum live shear in stringer.
(b) – Maximum dead shear in stringer.
(c) – Maximum live moment in stringer.
(d) – Maximum dead moment in stringer.
(e) – Total maximum shear in stringer (live and dead).
(f) – Total maximum moment in stringer (live and dead).
(g) – Maximum live shear in floor beam.
(h) – Maximum dead shear in floor beam.
(i) – Maximum live moment in floor beam.
(j) – Maximum dead moment in floor beam.
(k) – Total maximum shear in floor beam (live and dead).
(l) – Total maximum moment in floor beam (live and dead).

Note to Problem 74.—The maximum shear (both live and dead) on a simple beam always occurs at one end. For the maximum shear in the stringer BC, Fig. 93, the reaction at B and, hence, the shear at B will be a maximum when one of the heavy loads is at B with the other heavy loads on BC and as near to B as possible, irrespective of the loads on AB.

ASSIGNMENT 10

CHAPTER VIII

CONCENTRATED LOAD SYSTEMS—GRAPHICAL TREATMENT

49. Graphical Construction.—A graphical or equilibrium polygon method of determining maximum shears and maximum moments, due to a system of concentrated loads, is often used in practice. To be sure, the results obtained by such a method are not quite as exact as if computed by means of a moment-table, but they are often close enough for practical purposes. The algebraic or moment-table method is preferred by many engineers, but there seem to be a good number who make their calculations graphically.

Directions for making the necessary graphical construction in this method are as follows:

On a line *AB*, Fig. 94, and preferably to a scale of 10 ft. to an inch, mark the positions of the wheel loads and place points at 5-ft. intervals along the uniform load; the uniform load being assumed concentrated at these points. The loads, which are for one rail only, are generally given above the line and their summation below.

Next, construct the force polygon. The manner of doing this is as follows: On a vertical line *hk*, at the left of the plate, lay off the wheel loads successively to a convenient scale of pounds and then number these loads to correspond with those on the line *AB*. The *pole O* should be chosen to the right of the vertical and preferably about opposite its middle point and distant from it horizontally some even number of pounds, measured by the same scale as that used for the loads. The best results will be obtained when the *pole-distance* is equal to about one-quarter the length of the load-line. In the figure it is taken at 150,000 lb. From the pole, lines should be drawn radially to the points of division on the load-line.

The equilibrium polygon should now be constructed: Beginning at any point on the vertical line through wheel 1, draw a

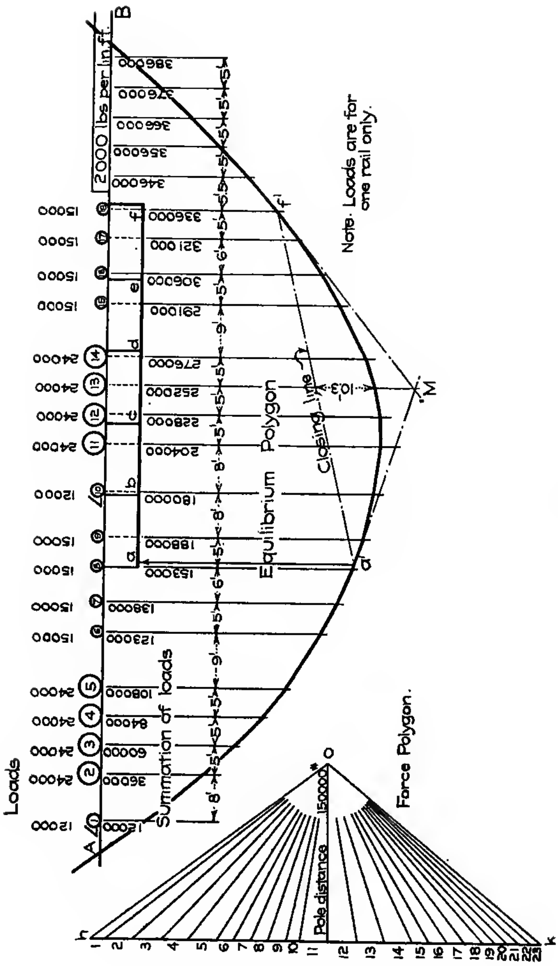
Loads
Summation of loads
Equilibrium Polygon
Closing line
2000 lbs per lin. ft.
Note. Loads are for one rail only.
Force Polygon
Pole distance

FIG. 94.

line of indefinite length to the left, parallel with the radial line which passes through the upper end of load 1 in the force polygon. From the same point on the vertical through wheel 1, draw a line to the right, parallel with the radial line which passes through the lower extremity of load 1 in the force polygon. Continue this line until it intersects a vertical through wheel 2. From this intersection draw a line to the right, parallel with the radial line which passes through the lower extremity of load 2 in the force polygon, and intersecting the vertical through wheel 3. In this manner all the *strings* of the equilibrium polygon are drawn. It should be observed that the lines drawn to the left

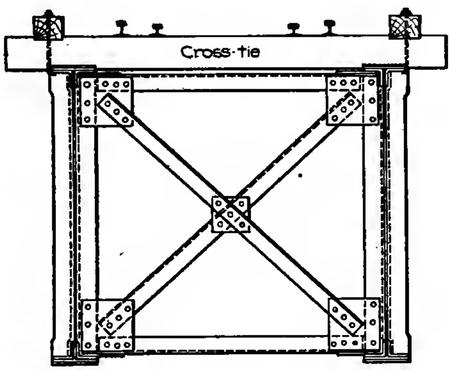

FIG. 95.

and right of a vertical through a wheel-concentration are parallel with the radial lines (or *rays*) which pass through the upper and lower extremities of the corresponding load in the force polygon. In order to insure accurate results, the force and equilibrium polygons should be constructed with great care on drawing paper, using fine ink lines.

50. Maximum Reaction.—Let us consider a girder bridge of the *deck* type, the ties resting directly on top of the girders as is shown in Fig. 95. Now suppose that the maximum reaction or end shear is required on such a girder (with span of 60 ft.) due to the system of loads shown in Fig. 87. A diagram of the girder should be constructed on tracing cloth, to the same scale as the equilibrium polygon, and divided into any number of equal spaces. We shall divide the girder in question into five equal parts. The tracing cloth should be large enough so

that the closing lines and other marks can be made on it and thus avoid marking on the force and equilibrium polygons.

Since every load on the span contributes to the end reaction, it follows that for the maximum reaction, the live load should cover the span with the heavier loads as near as possible to the end considered. Therefore, wheel 2 should be placed over the end *a*. The diagram of the girder should be brought to this position, as shown in Fig. 96, and verticals dropped from the ends *a* and *f* to the equilibiium polygon, intersecting it in the points *a′* and *f′*. Then a line drawn through these points is

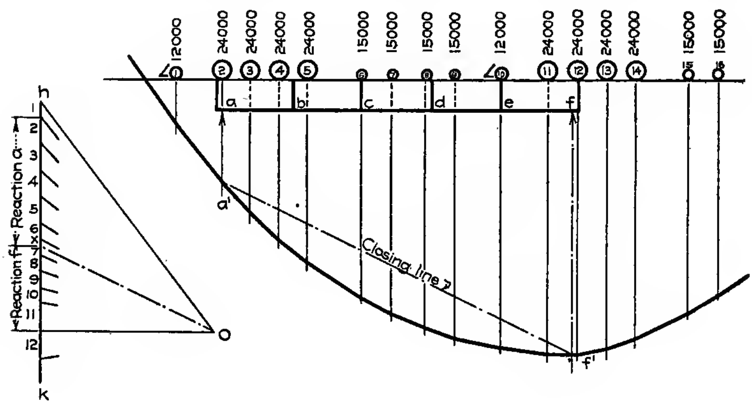

FIG. 96.

the closing line for that portion of the equilbrium polygon which belongs to the loads on the span; and a line drawn from the pole *O* in force polygon, parallel with the closing line, and intersecting the vertical load-line *hk*, determines the reactions at both ends. The upper part, which measures 116,000 lb., is the reaction at *a*; the lower part, the reaction at *f*, as shown. The reaction at *f* is the difference between the sum of loads 2 to 11 inclusive and the reaction *a*.

51. Maximum Shear Without Floor Beams.—Suppose the maximum shear is required at the section *b* of the girder. The diagram should be placed so that the wheel 2 is at the section *b*, Fig. 97. The verticals should be dropped from the ends *a* and *f* (as before) to the equilibrium polygon, and the closing line drawn. The left reaction is then determined in exactly the same manner as already described. The shear at section

b for this position of the loads is equal to the left reaction minus load 1 which acts to the left of b. From the study of influence lines it is clear that only the heaviest loads need to be tried at the section. Generally, the maximum will be found at either wheel 2 or wheel 3.

The above method is entirely graphical. Usually, it is found more convenient to determine the position of loads algebraically and then find the value of the shear for this position graphically.

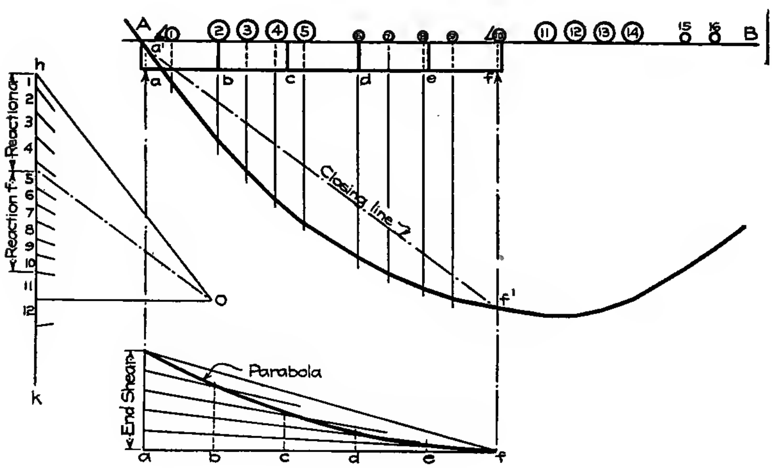

Fig. 97.

The position may be determined by the *gain or lose* method already described, but there is also another method used in practice with which the student should be familiar.

Suppose the maximum shear is required at any section on a girder without floor beams, such as section A, Fig. 83. Place some load just to the right of A, which for convenience we shall call P_1. Let G_1 then represent the sum of the loads to the left of, and including P_1, and G_2 the sum of the loads to the right of P_1. Also, let G equal the total load on the girder when P_1 is at A, and b the distance between P_1 and the next load to the right which we shall call P_2.

Now suppose the system of loads be moved a distance b to the left thus bringing P_2 to A. The effect upon the positive shear is

first to decrease it suddenly by an amount P_1, after which it is gradually increased. The increase due to G_2 may be expressed by

$$G_2\ b \tan \alpha \text{ (see influence line)}$$

and the increase due to G_1 (decrease in negative shear) may likewise be expressed by

$$G_1\ b \tan \alpha$$

The net change in shear due to the entire movement is:

$$G_1\ b \tan \alpha + G_2\ b \tan \alpha - P_1$$

$$\text{or } G \frac{b}{L} - P_1.$$

If this expression is positive then the second position gives the greater shear and, if negative, the first position. For equal shears we have, therefore,

$$\frac{G}{L} = \frac{P_1}{b}.$$

The slight increase in shear due to additional loads that may come upon the beam from the right has been neglected. The above expression means that to increase the shear we move to the left provided the average load per foot on the whole span is greater than the load at the section divided by the distance between this load and the next load to the right.

Suppose now that the maximum shear is required at section A of the 60-ft. span shown in Fig. 87a. The diagram of girder should be placed on the equilibrium polygon with wheel 1 at A. In this position the total load on the span equals 168 and the load at the section is 12. Then, $\frac{168}{60} > \frac{12}{8}$, and the shear is greater with wheel 2 at A. Now consider wheel 2 and wheel 3. Here the total load on the span equals 180 and the load at the section is 24. Then, $\frac{180}{60} < \frac{24}{5}$ and wheel 2 at A gives a maximum condition.

Since the slight increase in shear due to additional loads that may come upon the girder from the right has been neglected in deriving the above criterion for maximum shear, the effect of such loads must be investigated. If G' be the total load on the

girder when P_2 is at A, then the increase in shear when moving up P_2 will be somewhere between $G\frac{b}{L}-P_1$ and $G'\frac{b}{L}-P_1$. It may be possible for the first expression to be negative and the latter positive. Such a circumstance would result in causing $\frac{G}{L}$ to be less than $\frac{P_1}{b}$ for two succeeding loads and both positions would have to be tried. Such a condition is not likely to arise, however, when dealing with locomotive wheel loads since the loads do not advance to the left far enough at any one operation to so affect the criterion.

An approximate method may also be used to determine maximum shear at any point of a girder without floor beams. The method referred to gives sufficient accuracy in many cases, but the maximum end shear must first be determined. Construct a parabola with apex at f, Fig. 97, and the maximum ordinate at a equal to the end shear. The maximum live load shear at any point will then be the ordinate to the parabola at that point. A simple way in which a parabola may be constructed is indicated in the drawing.

52. Maximum Moment Without Floor Beams.—In connection with our study of the moment table we found that a maximum moment at any point occurs when some load lies at that point, and that load must be such that when it lies just to the right of the section, the average load on the whole span will be greater than the average on the left of the section, while if it lies to the left of the section, the average load on the left will be the greater. We make use of this fact in determining the position of loads for maximum moment by the graphical method in the same manner as when using a moment table.

Suppose the moment is required at section b with wheel 3 at the section. (This is the position of loads for maximum moment at b.) Fig. 98 shows the closing line of the equilibrium polygon for the reactions and loads on the span. The moment at b, then, is equal to the ordinate at b, measured vertically, between the closing line and the equilibrium polygon multiplied by the pole-distance in the force polygon. The reason for this statement is explained in Art. 38. The ordinate scales 6.3 ft. and the pole-distance is 150,000 lb. The maximum moment at $b=6.3\times150{,}000=945{,}000$ ft. lb.

If the *absolute maximum moment* is desired near the center of the girder, the greatest possible load should be placed on the span, with the heavier wheels near the center. Then, if the center of gravity of the total load on the span coincides with a wheel-concentration, the maximum moment will occur under this wheel, when placed at the center of span. If the center of gravity of the load falls between two wheels, the maximum moment will be under one of these (usually the wheel nearest the center of gravity), when the wheel considered and the center of gravity are equidistant from the center line of span. It may

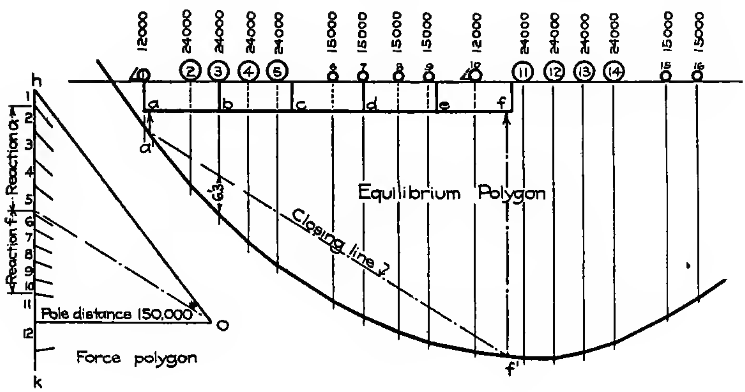

FIG. 98.

be necessary to try both of the wheels adjacent to the center of gravity to determine the maximum moment. The reason for the above procedure is explained in Art. 47.

With wheel 13 at the center, as shown in Fig. 94, it is evident that the greatest possible load is on the span. The center of gravity of loads 9 to 17 inclusive, is found by extending the outer sides of that portion of the equilibrium polygon which belongs to these loads to an intersection in the point M, which is directly below wheel 13; and, consequently, this load is at the center of gravity, and this position will give the absolute maximum moment. Wheel 18, which is at the point of support f, is not included in the above system of loads because it does not affect the bending moment. The ordinate at the center of span measures 10.3 ft.; then, 10.3x150,000 = 1,545,000 ft. lb. = absolute maximum moment.

53. Maximum Shear With Floor Beams.—The position of loads to give maximum shear in any given panel of a girder or truss must first be determined before the value of this maximum shear can be found. The position may be determined by the *gain or lose* method already explained, but there is also another method used in practice with which the student should be familiar.

Let Fig. 99 represent two locomotives and train on a bridge having floor beams. Suppose the maximum shear from the live load is required in panel *bc*. Let G_1 be the total load on the

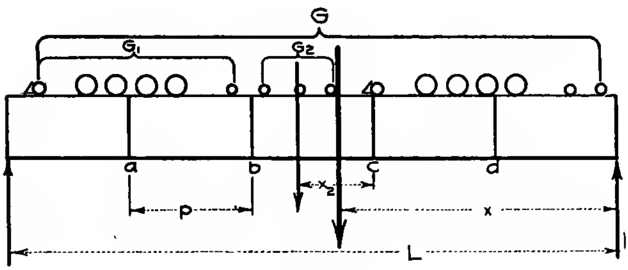

FIG. 99.

bridge to the left of the panel in question, G_2 the sum of the loads in the panel *bc*, and G the total load on the span. Also let x equal the distance from G to the right abutment, and x_2 the distance from G_2 to the point c.

Then the shear

$$V=\frac{Gx}{L}-\frac{G_2x_2}{p}-G_1.$$

Let the system of loads be moved a distance $\triangle$ to the left; then the new shear is

$$V'=\frac{G(x+\triangle)}{L}-\frac{G_2(x_2+\triangle)}{p}-G_1$$

The shear has been increased by the operation, provided

$$\frac{G(x+\triangle)}{L}-\frac{G_2(x_2+\triangle)}{p}-G_1>\frac{Gx}{L}-\frac{G_2x_2}{p}-G_1 \quad \text{or} \quad \frac{G}{L}>\frac{G_2}{p}.$$

The above expression means that to increase the shear we move to the left if the average load per foot on the whole span is

greater than the average load in the panel in question, and vice versa. Hence, we find that the maximum shear in the panel will occur when some load is at the panel point at the right of the panel, and that load must be such that when it lies just to the right of the panel point, the average load on the whole span will be greater than the average in the panel, while if it lies to the left of the panel point, the average load in the panel will be the greater. More than one maximum may be found under each set of heavy loads.

Suppose now that the maximum shear is required in panel bc of the truss shown in Fig. 100. The diagram of truss should be placed on the equilibrium polygon with wheel 2 just to the right of c, as shown. In this position, the load in the panel $bc=12$, and the total load on the span equals 204; which figures are obtained from the summation of loads shown below the wheel loads. Then, $\frac{204}{5p} > \frac{12}{p}$. With wheel 2 just to the left of c, the load in the panel $bc=36$; and the total load on the span is the same as before; namely, 204. Then, $\frac{204}{5} > 36$. Not a maximum. With wheel 3 just to the left of c, the load in the panel $bc=60$, and the total load equals 228. Then $\frac{228}{5} < 60$ and one condition for maximum shear is obtained with load 3 at c.

Fig. 100 shows the manner of obtaining the value of the shear with load 2 at c. The same method would be used with any load at the floor beam to the right of the panel. The left reaction which is determined is greater than the shear in bc by the amount of the loads transmitted to panel point b, or in this case by $\left(\frac{8}{20}\right)(12)$. Thus, the shear in $bc = 77{,}800 - 4800 = 73{,}000$ lb.

The amount to be subtracted can be determined graphically, by drawing the closing line of the equilibrium polygon for the span $b'c'$. A line drawn from O in the force polygon parallel to this line determines the reaction at b, as shown.

54. Maximum Moment with Floor Beams.—As already explained, it is only necessary to determine the maximum moments at floor beams and to do it the same as if there were no floor beams.

Problem 75.—(a) Find the position of the live loads shown in Fig. 87 that will cause maximum positive shear at sections a, b, and c of the girder shown in Fig. 80. Consider only the heavy loads of the first locomotive. Use the algebraic method of this assignment. (b) Determine the value of the maximum shear at points a, b, and c by the equilibrium polygon method.

Problem 76.—(a) Determine the maximum moment at sections a, b, and c of the girder shown in Fig. 80 for the loading shown in Fig. 87. Use the

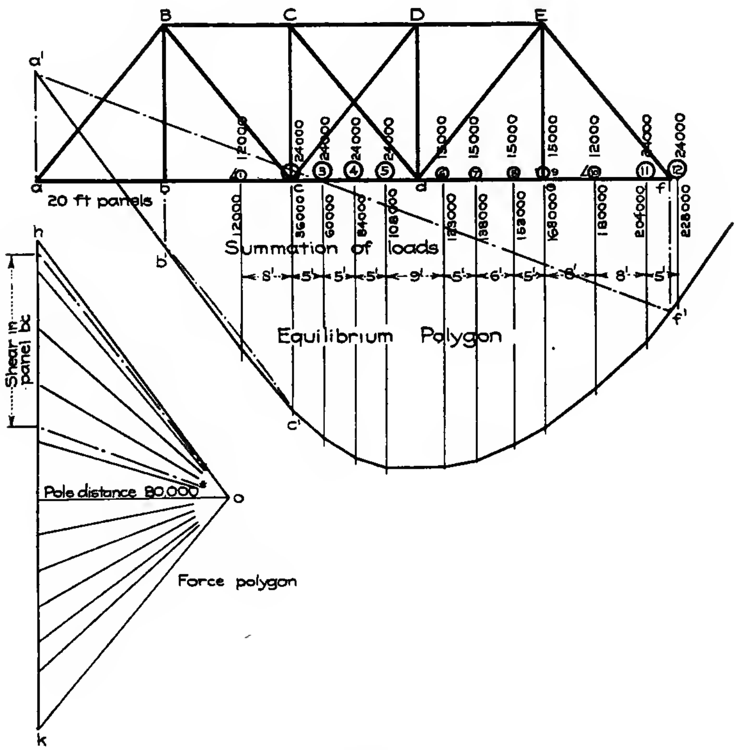

FIG. 100.

equilibrium polygon method. (b) Determine the absolute maximum moment.

Problem 77.—Consider the 40 ft. girder shown in Fig. 80 to have 4 equal panels. Loading as shown in Fig. 87. (a) Determine the position of loads to give maximum positive shear in the first and second panels from the left support, using the method of Art. 53. (b) Obtain graphically the value of the maximum shear in each panel for the position of loads determined in part (a).

Problem 78.—With the girder and loading as given in Problem 77, deter-

mine the maximum moment at panel points b and c. Equilibrium polygon method.

Problem 79.—(a) A truss has 7 panels at 15 ft. = 105 ft. span. With Cooper's Class E 50 shown in Fig. 11 obtain graphically the maximum shear in the second panel from the left support. Obtain the position of loads for a maximum, algebraically. (b) With the same truss and loading, obtain the maximum moment graphically at the fourth panel point (not including the panel point over the end reaction) from the left end.

ASSIGNMENT 11

CHAPTER IX

STRESSES IN TRUSS MEMBERS

[The work thus far in the course has been, for the greater part, the determination of the outer forces acting upon structures, and the methods used in finding maximum shears and maximum moments. The student should understand from the course in "Strength of Materials" why maximum shears and maximum moments are needed in the design of simple beams. They are used in the design of girders in much the same manner. There still remains to be explained how these maximum functions of the outer forces are employed in the design of trusses. It should be remembered that shear is constant in each panel for a given loading and that the moments need be computed only at floor beams.]

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55. General Methods of Computing Stresses.—Only the general methods of computing stresses in trusses will be given, in order to clear up in the mind of the student the reason why maximum shears and maximum moments for trusses were previously computed. The methods *in detail* for finding stresses in different trusses are completely treated in "Bridge Trusses, Part 1," Course 412.

The general methods of computing stresses in trusses are:

1. Method of sections.
2. Method of joints.

The method of joints is in reality a method of sections, but it is so distinct in its treatment from the so-called method of sections that we shall consider it a method by itself. It is very seldom that either method is alone used to compute all the stresses in a given truss. It is generally found more convenient to compute the stresses in some members of a truss by one method and the stresses in the remaining members by the other method.

In either method the necessary procedure, in order to determine stresses for a given loading, is to separate the given truss into two parts by an imaginary section, either plane or curved; the part of the truss to one side of the section is removed (that is, considered so) together with all external forces, and the bars that are cut by the section are replaced by the stresses acting in those bars. By so doing, the part of the truss which we are

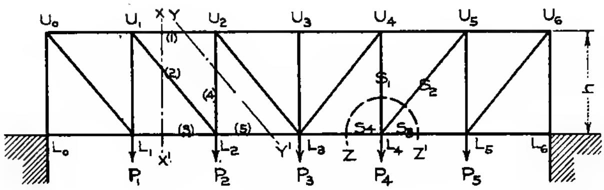

FIG. 101a.

considering will be in equilibrium due to the outer forces acting on that portion of the truss and the stresses in the bars cut. If the section is taken completely across the truss as XX' or YY', Fig. 101a, so that the members cut *do not* all intersect in one point, then the method used is the *method of sections.* If the section is so taken that the members *do* all intersect in one point as ZZ', Fig. 101a, then the method used is the *method of joints.* (Fig. 101c.)

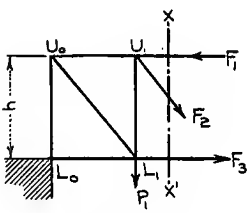

FIG. 101b.

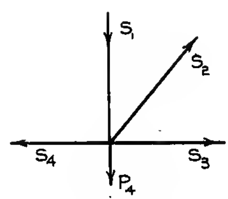

FIG. 101c.

56. Algebraic Treatment.—Let us first consider the algebraic treatment of the *method of sections* as applied to a bridge truss. Required the maximum stresses in bars (1), (2), and (3) of the truss shown in Fig. 101a, these bars being cut by the section XX'. Consider the portion of the truss shown in Fig. 101b. For a definite loading the forces are all in equilibrium as explained above and, since only three bars are cut, we can use any

or all of our three equations of equilibrium; namely, $\Sigma H=0$, $\Sigma V=0$, and $\Sigma M=0$. We shall first use the equation $\Sigma M=0$. This equation is true about any point in the plane of the truss but, in order to get the stress in a given bar directly, we must take the center of moments at the intersection of the other two bars. For example, the stress in F_3 for a given loading can be found by taking moments about the point U_1. It should be noticed that U_1 is vertically above L_1 and, since the loads are all vertical, the moments at U_1 and L_1 are equal. The *maximum* stress in F_3, then, occurs with the loading which gives maximum moment at the first panel point from the left support. Call this maximum moment M_1. The moment of F_3 (when F_3 is a maximum) about the point U_1 must be equal and opposite to M_1 in order that ΣM may equal zero. Thus,

$$(\text{max. } F_3)\ (h)=M_1$$

$$\text{or max. } F_3=\frac{M_1}{h}.$$

In the same manner, calling M_2 the maximum moment at the second panel point,

$$\text{max. } F_1=\frac{M_2}{h}.$$

The student should observe (using $\Sigma M=0$) that the stress in the upper chord acts toward the section, thus denoting compression, while the stress in the lower chord acts away from the section, thus denoting tension; that is, $F_1=$ compression and $F_3=$tension. This is true of all the upper and lower chords throughout the truss.

The maximum stress F_2 remains to be found. We may do this by the equation $\Sigma V=0$. The vertical component of the maximum stress in F_2 is equal to the maximum positive shear in the second panel from the left support. Call this component V_2.

Then,

$$\text{max. } F_2=V_2\frac{U_1L_2}{h}.$$

In using the equation $\Sigma V=0$, the student should observe that the stress acts away from the section, thus denoting tension.

Let the maximum stress be required in bars (1), (4), and (5), Fig. 101a. Take the section YY'. Using $\Sigma H=0$, and

knowing that the loads are all vertical, the stress in bar (1) is seen to be equal and opposite to the stress in bar (5). This applies for any loading, hence, the loading giving maximum stress in bar (1) will also give a maximum stress in bar (5) of the same amount; that is, the loading giving the maximum moment at the second panel point from the left support will cause maximum stress in both bars (1) and (5). The maximum stress (compression) in bar (1) is, as before, $\frac{M_2}{h}$ using $\Sigma H=0$. This same amount of tension, then, occurs in bar (5). The maximum stress in bar (4) is directly the maximum positive shear in the third panel from the left support, using the equation $\Sigma V=0$. Stress in bar (4) is compression.

In the method of sections, the section should always be taken so as to cut only three bars whose stresses are unknown. If more than three bars are cut, we have more unknown quantities than can be found by the principles of statics.

The method of joints is only a name given to the manner of determining stresses from the conditions of equilibrium of concurrent forces. The manner of using the algebraic conditions, namely, $\Sigma H=0$ and $\Sigma V=0$, was explained in Art. 26 by determining the stresses in the members of a crane truss (which see). It should be clear that this method can be applied to a joint only when there are two unknown stresses. In solving a truss by this method, it is evident that we must begin with a joint where but two bars meet and proceed from this to other joints. The student should study Fig. 101a and see how, by commencing at the abutment, the stresses in all the members of the truss may be worked out for any given loading.

In the algebraic method of joints, if a maximum stress is desired in a certain member of a truss, all the joints from one end of the truss up to the member considered must be computed for the loading giving maximum stress in that member only. For this reason the algebraic method, although perfectly general, is too laborious to be employed in practice in determining the maximum stresses in all the bars of an ordinary truss. It may be used with great advantage, however, for certain specific members and should be understood. A graphical method based upon the same principles is well adapted for many types of trusses, particularly roof trusses with non-parallel chords. In roof trusses, the conditions for probable maximum stress in the

given members are few, and usually all the stresses may be computed graphically for each loading in much shorter time than it would take to compute the stresses throughout the truss algebraically for any *one* condition of loading.

Illustrative Problem.—Roof truss of Fig. 102a; loads as shown. (a) Required the stresses in all members algebraically by the method of sections. (b) By the method of joints.

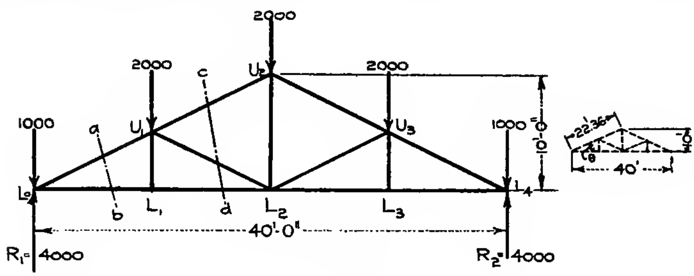

FIG. 102a.

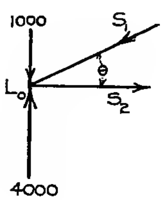

FIG. 102b.

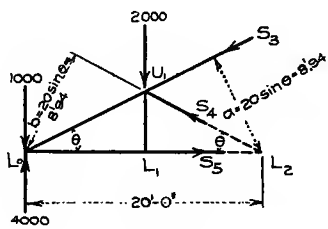

FIG. 102c.

(a) METHOD OF SECTIONS

To find the stresses in members L_0U_1 and L_0L_1 pass a section a–b cutting these members. Consider the truss to the left of the section. We have a case of concurrent forces at the joint L_0. Fig. 102b shows the joint at L_0 removed and the known loads applied, together with the unknowns S_1 and S_2, assumed to act as shown. Consider upward forces and forces to the right as positive; downward forces and forces to the left as negative. The two equations, $\Sigma V=0$ and $\Sigma H=0$, may be employed to find the two stresses S_1 and S_2.

$\Sigma V=0.\quad 4000-1000-S_1\sin\theta=0$

$S_1=(3000)\left(\frac{22.36}{10}\right)=6710$ lb. (compression as assumed, since result is positive)

$\Sigma H=0.\quad S_2-S_1\cos\theta=0$

$S_2=(6710)\left(\frac{20}{22.36}\right)=6000$ lb. (tension, as assumed, since result is positive)

To find the stresses in members U_1U_2, U_1L_2 and L_1L_2 pass a section *c–d* cutting these members and consider the portion of the structure to the left (Fig. 102c). We have here a system of non-concurrent forces in equilibrium. The three equations of equilibrium may be used to determine the three unknown stresses, but the solution may be simplified by employing only $\Sigma M=0$ three times. This equation should be applied at the intersection of two bars to find the stress in the third. Thus, to determine the stress in U_1U_2, take moments about L_2, the intersection of U_1L_2 and L_1L_2. Then considering clockwise moments as positive,

$$4000\ (20)-1000\ (20)-2000\ (10)-S_3(a)=0$$
$$S_3=4470 \text{ lb. (compression)}$$

The stress in S_4 may be obtained by taking moments about L_o, the intersection of U_1U_2 and L_1L_2.

$$2000\ (10)-S_4(b)=0$$
$$S_4=2240 \text{ lb. (compression)}$$

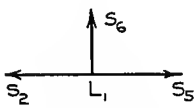

FIG. 103a.

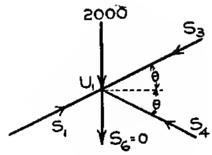

FIG. 103b.

The stress in S_5 may be found by taking moments about U_1, the intersection of U_1L_2 and U_1U_2.

$$(4000-1000)\ (10)-S_5(5)=0$$
$$S_5=6000 \text{ lbs. (tension)}$$

Other sections should now be taken cutting only three bars whose stresses are unknown and the moment equation again applied. Proceeding in this manner the stresses in all the members may be determined.

(b) METHOD OF JOINTS

The stresses in members L_0U_1 and L_0L_1 are determined as for the method of sections and the solution will not be repeated here.

Passing now to the next joint at which only two unknowns exist, we will select joint L_1, shown in Fig. 103a.

$$\Sigma V=0. \quad S_6=0$$
$$\Sigma H=0. \quad S_6-S_2=0$$
$$\text{or } S_5=S_2=6000 \text{ lb. (tension)}$$

Next pass to joint U_1, which is shown in Fig. 103b. The two unknown forces are S_3 and S_4.

$$\Sigma V=0. \quad S_1 \sin\theta+S_4 \sin\theta-S_3 \sin\theta-2000=0$$
$$S_4 \sin\theta-S_3 \sin\theta=-1000$$
$$\Sigma H=0. \quad S_1 \cos\theta-S_4 \cos\theta-S_3 \cos\theta=0$$
$$S_4 \cos\theta+S_3 \cos\theta=6000$$

These independent equations involve only the unknowns S_3 and S_4. Solving simultaneously, we have

$$S_4 - S_3 = -2236$$
$$S_4 + S_3 = +6708$$
$$\therefore S_3 = 4470 \text{ lb. (compression)}$$
$$S_4 = 2240 \text{ lb. (compression)}$$

The stresses at joint U_1 are now completely determined. In the same way pass to the other joints until all the stresses in the members of the truss are determined.

Problem 80.—Consider the truss of Fig. 101a to have a span of 120′-0″, with six equal panels, and h = 24 ft. Take the live load at 2000 lb. per ft. per truss and the dead load at 900 lb. per ft. per truss. Determine the *maximum* stress algebraically in bars (1), (2), (3), (4), and (5) by the method of sections.

Problem 81.—(a) Find the stress algebraically in the center member of the lower chord of the roof truss shown in Fig. 9 by the method of sections. Consider the roof truss subjected only to the loads shown. (b) By the method of joints find the stress algebraically in all the members of this truss.

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57. Graphical Treatment.—In the graphical method of sections we commence at one end of the structure and pass a section cutting but *two* members. The stresses in these members can be determined by the single condition that the force polygon, drawn for the forces on one portion of the structure, must close. Next a section is taken cutting *three* members, one of which has already been determined, and the two unknowns can be found by the force polygon method as before. By successive sections taken in this manner, we can determine all the stresses by simple force polygons.

The graphical construction resulting from the method of joints is identical with that resulting from the method of sections. The only difference is the sections taken and, consequently, the order in which the lines are drawn. The method of joints is generally preferred in practice on account of its simplicity and this method only will be illustrated here.

Illustrative Problem.—Required the stresses in all members of the roof truss shown in Fig. 104a by the graphical method of joints; loads as shown.

It will simplify matters to draw a sketch of the truss to some suitable scale and show on it all the outer forces including reactions. Also, to desig-

nate all the forces and bars on this sketch by letters so located that each force and each bar will lie between two letters and only two, as illustrated in Fig. 104a.

Now any force as AB, for example, in this figure may be designated in the graphical solution by a line having a length corresponding to the magnitude

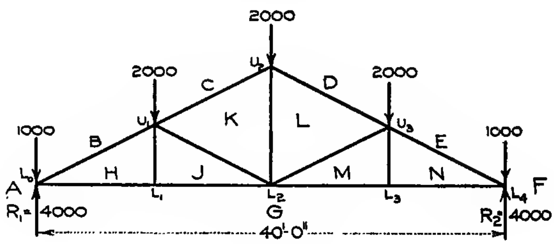

FIG. 104a.

of the force and with the letter A at one end and the letter B at the other. By going through the graphical construction in this manner one letter only need be placed at each apex of a force polygon and the work is greatly simplified.

The next step is to draw a force polygon for the outer forces to a scale of sufficient size to give the desired accuracy. The force polygon is $ABCDEF$-

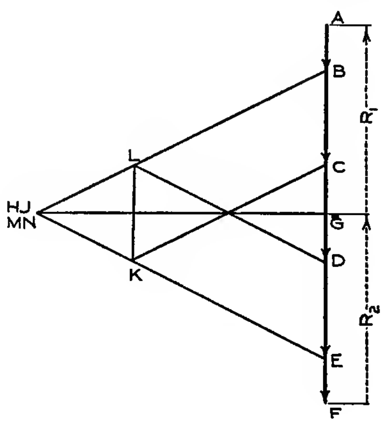

FIG. 104b.

GA in Fig. 104b and is a straight line, since all the forces are vertical. The external forces should be plotted in the order obtained by going around the figure in a clockwise direction. $AB=1000$. $BC=CD=DE=2000$. $EF=1000$. $FG=R_2=4000$. $GA=R_1=4000$. The right and left reactions must previously be computed either algebraically or graphically.

The triangle of forces should now be drawn for joint L_0. The unknown

forces which act at this joint are the stress in the bar BH and the stress in the bar HG. BH and HG are known in direction but not in magnitude, hence, there are but two unknowns and these can be found by the polygon of forces. The figure $ABHGA$, Fig. 104b, is this polygon obtained by drawing from B a line parallel to the bar BH, and from G a line parallel to HG. The lines BH and HG may now be scaled from the force polygon to obtain the magnitude of the stresses in the two bars intersecting at L_0. The character of these stresses must also be found The forces at joint L_0 being in equilibrium must follow in order around the corresponding force polygon. Reading around joint L_0 in a clockwise direction gives BH acting downward to the left, or toward the joint L_0, thus showing compression, and HG acting toward the right, or away from the joint L_0, showing tension.

The joint L_1 is the next one at which only two unknowns exist. The stress in GH is known from joint L_0, and HJ and JG are unknown. The corresponding force polygon HJG for this joint must close. Since GH and JG have the same line of action, the line in the force polygon representing the magnitude of HJ will be a point, thus having no length. The stress in HJ is, therefore, zero. This might have been seen by inspection, as there is no load at L_1 to cause stress in this member. In reading around joint L_1 in a clockwise direction, the line JG is from left to right, and the stress acts away from joint L_1 denoting tension.

Now pass to joint U_1. The stresses in bars CK and KJ are the unknowns. To obtain them draw CK and JK in the force polygon parallel respectively to the corresponding bars in the truss. (The stress being zero in JH, the whole space occupied by J and H may conveniently be called J.) Reading around joint U_1 in a clockwise direction gives both CK and KJ acting toward the joint U_1, hence, denoting compression in both these bars. The polygon considered is $BCKJB$. In a similar manner the stresses in the other bars may be determined.

Problem 82.—By the graphical method of joints determine the stresses in all the bars of the truss shown in Fig. 9 for the loading shown. Use scale of sufficent accuracy to give the stresses to the nearest 10 lb.

---

59. Statically Determinate Trusses.—Consider the truss of Fig. 101a. There are 14 joints and we have two equations of equilibrium for each joint, or 28 equations in all. These 28 equations must suffice to determine the unknown reactions as well as the unknown stresses in the bars, since equilibrium of each joint involves equilibrium of the structure as a whole. Now, in Art. 31, it is shown that a structure which is statically determinate with respect to the outer forces has three unknown conditions to be determined by the principles of statics. This leaves 25 equations by which to determine the stresses in the truss members; in other words, if the structure we are con-

sidering has no greater than 25 bars the truss is statically determinate with respect to both outer and inner forces, or is known simply as statically determinate. The truss we refer to is seen to have just this number and all the inner forces may thus be found.

In general it may be said that if n is taken to denote the number of joints of a given truss which is statically determinate with respect to the outer forces, then the truss in question must have just $2n-3$ bars in order for it to be statically determinate as a whole. If it has more bars, the stresses cannot be computed by statics, and if it has a less number, it is unstable and will collapse except under some particular loading.

ASSIGNMENT 12

CHAPTER X

DESIGN OF STRUCTURAL MEMBERS

59. General Discussion.—The inner forces were defined in Art. 4 as "the stresses in the different members of a structure which are called into action by the outer forces and by means of which the structure carries its load." The inner forces may be either tension, compression, or shear, or there may be a combination of these. A structural member subjected only to tension is known as a *tie;* one subjected only to compression is called a *column* (*strut*); while a member exposed simply to bending is either a *beam* or a *girder.* The inner forces in beams and

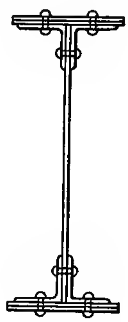

Fig. 105a.

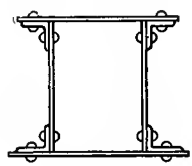

Fig. 105b.

girders are tension, compression, and shear. Although the general term *beam* may well apply to both beams and girders (see definition in Art. 1), the term girder as used in this chapter refers to the built up steel shapes known as plate girders and box girders. Fig. 105a shows the cross-section of a typical plate girder and Fig. 105b the cross-section of a typical box girder. Plate girders are used for lengths greater than are admissible for I–beams or where I-beams of sufficient strength cannot be obtained. A box girder is used in situations where great strength with limited depth is required. The design of

plate and box girders will not be considered in this course. The design of reinforced concrete members will also be omitted. Plate girders will be fully treated in "Plate Girder Bridges," Course 411 and in "Steel Building Construction," Course 421. Reinforced concrete members will be dealt with in "Reinforced Concrete Construction," Course 418.

The members of a truss are generally subjected only to direct stress; consequently, for such a condition, a truss is composed entirely of ties and struts. In many cases, however, the loads are not all applied at the joints and the members supporting transverse loads are subjected to bending in addition to direct stress. The general formulas for computing the stresses in such members were developed in the course in "Strength of Materials" and the application of these formulas to definite cases will be taken up in the courses which follow. Transverse flexure in addition to direct stress is always present in a horizontal strut, such as a bridge chord, where the dead weight of the strut causes bending moment. Usually, however, this moment is very small and is neglected. A good illustration of combined flexure and direct stress occurs in the upper chord of roof trusses when a roof covering of corrugated iron is used. (Figs. 3c and 3d). Another example occurs in the top chord of a deck bridge truss when it is used to support the track ties directly.

In Chapter IX it has been shown in general how the maximum stresses in the different members of a truss may be found. If the truss is already in existence, then the maximum stress in a given tension or compression member divided by its smallest cross-sectional area gives the maximum stress per square inch in that member. This intensity of stress should *not* be greater than the allowable stress per square inch, otherwise the structure should be considered unsafe. If, on the other hand, the structure is to be built, then the maximum stress in a given member divided by the allowable stress per square inch gives the minimum cross-sectional area required. If the member is a tie, then the *net* area of cross-section should be considered; if a strut, the *gross* area should be used. That is to say, any open holes in a compression member cause a diminution of area and must be allowed for, but if a hole is made and filled up by a rivet, the hole need not be deducted in proportioning the cross-sectional area. The above method of procedure may be followed for any

structural member of homogeneous material subjected simply to tension or compression as, for example, a symmetrically loaded column in a building.

The design of a compression member must be made by trial. The allowable average compression per square inch on a column depends upon its length and its least radius of gyration. This allowable unit stress may be computed by a column formula for any assumed cross-sectional dimensions and then, if the computed allowable stress varies considerably from the stress the column actually should carry, a new cross-section should be assumed based on the first computations. One or two trials should be sufficient to determine the column section best adapted to the given case.

60. Beam Design.—The student has had considerable practice in the designing of homogeneous beams in the course on "Strength of Materials." The following principles and formulas should be familiar:

1. At any cross-section the internal forces or stresses may be resolved into normal and tangential components. The components normal to the section are stresses of tension and compression, while the tangential components add together and form a stress known as the resisting shear.

2. The shear at any cross-section is borne by the tangential stresses in that section. The moment at any section is borne by the component stresses normal to that section.

3. The neutral axis passes through the center of gravity of the cross-section.

4. The intensity of stress normal to the section increases directly with the distance from the neutral axis and is a maximum at the extreme fiber. This maximum intensity for either tension or compression may be expressed by the formula

$$p=\frac{MX_1}{I} \tag{1}$$

where p = stress in extreme fiber of section in pounds per square inch.

M = external bending moment at section in inch pounds.

X_1 = distance in inches from neutral axis to extreme fiber.

I = moment of inertia of the section about the neutral axis.

5. The general formula which gives the longitudinal shear per square inch at any desired point in the cross-section is

$$v=\frac{VQ}{Ib'} \tag{2}$$

in which

v = intensity of the longitudinal shear per square inch at the desired point in the cross-section.
V = total external shear at the section in pounds.
Q = statical moment about the neutral axis of that portion of the cross-section lying either above or below (depending upon whether the point in question is above or below the neutral axis) an axis drawn through the point in question parallel to the neutral axis.
I = as before.
b' = width of beam at the given point.

In the above formula, by the term *statical moment* is meant the product obtained by multiplying the area above mentioned by

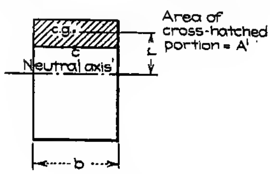

FIG. 106.

the distance between its center of gravity and the neutral axis. For example, the longitudinal shearing intensity at a point c in a rectangular beam, Fig. 106, may be expressed as follows:

$$v=\frac{VA'r}{Ib}$$

For rectangular beams and all beams of uniform width the largest value of v for any given section will occur at the neutral axis since the statical moment Q has its maximum value for a point on this axis and b is constant.

6. If a beam is of constant cross-section throughout, the maximum values of p and v will occur at the section where M and V respectively have maximum values.

The notation given in the Cambria handbook on page 144 will be followed. (The different tables and formulas are not given on the same pages in the different editions. For example, the notation mentioned above may be on page 140 in one edition and on page 149 in another. All the pages mentioned in this course will refer to the 1909 edition.)

In addition to the notation given in Cambria on page 144, b and d will be used to represent respectively the width and depth of rectangular beams.

An important consideration in selecting beams is that concerning vertical and horizontal stiffness. The subject of stiffness will be treated later, it being assumed for the present, that the beams designed in this chapter are supported laterally where necessary, and that their vertical deflection is not excessive.

Wooden Beams.—Wooden beams are generally rectangular in section, hence, for designing such beams our flexure formula may be more conveniently used in the form

$$M = \frac{1}{6}(p)(bd^2) \tag{3}$$

This form may be derived from the expression $p = \frac{MX_1}{I}$ as follows:

$$I = \frac{1}{12}\, bd^3 \qquad X_1 = \frac{d}{2}$$

Substituting, $$p = \frac{M\left(\frac{d}{2}\right)}{\frac{1}{12}\,(bd^3)} = \frac{6M}{bd^2}$$

$$\text{or } M = \frac{1}{6}\ (p)\ (bd^2)$$

The expression for the longitudinal shear v at the neutral axis may be simplified, as follows:

$$v = \frac{V\,b\left(\frac{d}{2}\right)\left(\frac{d}{4}\right)}{\left(\frac{1}{12}\,bd^3\right)b} = \frac{3}{2}\ \left(\frac{V}{bd}\right) \tag{4}$$

The commercial sizes of wooden beams are given in the following table:

• COMMERCIAL SIZES OF YELLOW PINE •

2 x 3	2 x 4	2 x 5	2 x 6	2 x 7	2 x 8	2 x 10	2 x 12	2 x 14	2 x 16
	3 x 4		3 x 6		3 x 8	3 x 10	3 x 12	3 x 14	3 x 16
	4 x 4		4 x 6		4 x 8	4 x 10	4 x 12	4 x 14	4 x 16
			6 x 6		6 x 8	6 x 10	6 x 12	6 x 14	6 x 16
					8 x 8	8 x 10	8 x 12	8 x 14	8 x 16
								10 x 14	10 x 16
								12 x 14	12 x 16
								14 x 14	14 x 16
									16 x 16

Note: Commercial sizes of spruce to the left of heavy line only.

Lengths of yellow pine sticks run up to 40 ft. and except for the largest sizes 50-ft. lengths may usually be obtained. For spruce, 12 to 22 ft. are ordinary lengths while up to 32 ft. may be obtained with difficulty.

The cost of wooden beams depends upon the price of lumber per board foot so that a beam having the smallest cross-sectional area is the cheapest. The *board foot* is the contents of a board 1 ft. square and 1 in. thick.

Illustrative Problem.—Design wooden beams to span 20 ft. and to carry a total uniform load (live and dead) of 3000 lb. per ft. The allowable stresses are to be taken as follows:

Bending, extreme fiber stress = 1000 lb. per sq. in.
Shearing along grain = 100 lb. per sq. in.
Bearing across grain = 250 lb. per sq. in.

(No allowance for impact need be made in wooden beams owing to the great elasticity of the wood.)

Maximum moment occurs at the center of beam and equals (Art. 41).

$$\frac{1}{8}\ (3000)\ (20)\ (20)\ (12) = 1{,}800{,}000 \text{ in. lb.}$$

Maximum shear is at the end of beam and equals

$$3000\ \left(\frac{20}{2}\right) = 30{,}000 \text{ lb.}$$

$$M = \frac{1}{6}\,(p)\,(bd^2)\ \therefore\ 1{,}800{,}000 = \frac{1}{6}\,(1000)\,(bd^2)$$

$$bd^2 = 10{,}800$$

Assume the depth required to be 16 in., then $b = 42.2$ in. and 3 – 14x16 in. sticks will be satisfactory, as regards bending.

The area of cross-section required to carry longitudinal shear may be determined by Eq. 4 and is as follows:

$$bd = \frac{3V}{2v} = \frac{(3)\ (30{,}000)}{(2)\ (100)} = 450 \text{ sq. in.}$$

Evidently the 3 beams selected for bending are of sufficient size to provide properly for longitudinal shear. The shearing strength generally controls in the case of comparatively short beams heavily loaded.

The bearing area on the abutment should now be determined. If the reactions were distributed uniformly over the supports, the bearing surface needed would be $\frac{30{,}000}{220} = 120$ sq. in. To provide for unequal distribution due to the cause explained in Art. 32, 50 per cent. will be added to this, giving 180 sq. in. The 14x16-in. beams should therefore extend $\frac{180}{42}$ or, say 4 1/2 in. over the abutment at each end.

Steel Beams.—I-beams are the most commonly used steel beams. In selecting such beams the sections in the handbook marked *standard* should be chosen since the selection of a *special* section is likely to cause delay in filling the order. The cost of a steel beam depends principally upon its weight.

The size and weight of an I-beam required can be taken from the handbook if the section modulus $\frac{I}{X_1} = S$ is known. The method in the design of I-beams is thus to ascertain the necessary section modulus after assuming the dead weight of beam. The size of beam required may then be taken from the handbook. If the weight of the I-beam chosen differs very materially from the assumed weight, the section modulus should be recomputed and the size of beam again found. This, however, is seldom necessary since the moment and shear due to the weight of beam is a very small percentage of the total moment and shear.

Illustrative Problem.—Design steel beams to span 15 ft. and to carry a live uniform load of 5000 lb. per ft. The allowable stresses are to be taken as follows:

Extreme fiber stress in bending $= 16{,}000$ lb. per sq. in.

Shear on net section of web $= 12{,}000$ lb. per sq. in. Fifty per cent. of total load to be added to allow for impact.

Maximum moment is at the center of the beam, and assuming the dead weight at 80 lb. per ft.

$$M_{max} = \frac{1}{8}\ (5080)\ (15)\ (15)\ (12)\ (1.5) = 2{,}571{,}750 \text{ in. lb.}$$

$$\therefore \frac{I}{X_1} = \frac{2{,}571{,}750}{16{,}000} = 161.$$

Refer now to column 8, page 168, of the handbook. A 24-in. I-beam, weighing 80 lb. per ft. has sufficient strength and will be chosen. Two 18-in. 55-lb. beams are also strong enough, but are more expensive than the 24-in. beam. Conditions, however, might require a shallower beam.

It will be shown later that the shear on an I-beam is practically all carried by the web over which it is distributed almost uniformly. Making this assumption the area required in the web would be $\frac{(5080)(15/2)(1.5)}{12,000} = 4.8$ sq. in. The actual area in the beam selected is far in excess of this and the beam is hence strong enough to carry the shear.

The student should now be able to compute any ordinary wooden or steel beam. Channels and angles are often used as beams and their design is similar to that of I-beams. For channels and angles see pages 170 to 187 inclusive.

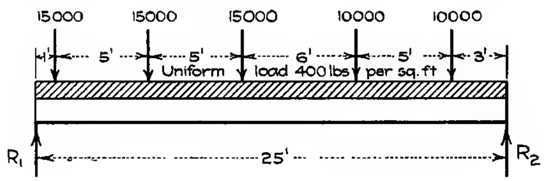

FIG. 107.

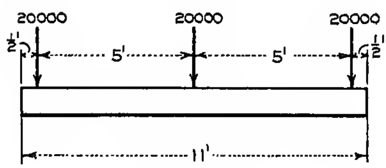

FIG. 108.

The method used in designing beams may be stated in general as follows: determine the loads, compute the reactions, compute the maximum moment, find the resulting section modulus, and then look up the necessary dimensions in the handbook.

Problem 83.—Design a wooden beam to span 12 ft. and to carry a total uniform load (live and dead) of 1200 lb. per ft. Use allowable stresses of illustrative problem.

Problem 84.—Design an I-beam span of 25 ft. to support the fixed loads shown in Fig. 107. Take the allowable stresses as follows:

Extreme fiber stress in bending = 10,000 lb. per sq. in.

Shear on net section of web $=7500$ lb. per sq. in. and do not allow for impact.

Problem 85.—Design wooden and steel stringers for a single-track railroad bridge. Panel length 11′–0″. Fig. 108 shows wheel loads which will cause maximum moment, the maximum moment occurring at the center of the beam. Wooden beam is not to exceed 16 in. in depth. Use only *standard* I-beams. Assume allowable stresses same as in Problem 84. The dead weight of track is 400 lb. per ft. per track or 200 lb. per ft. per rail.

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61. Design of Tension Members.—The design of a tension member consists simply in selecting a bar with sufficient net area to carry the total stress without exceeding the allowable unit stress. Iron and steel tension members are of two general types: (1) solid bars either circular or rectangular in cross-section, and (2) built up members composed of structural shapes riveted together. Solid bars are generally used in pin trusses for diagonals and bottom chord members. Built up tension members are generally employed in riveted trusses and for the end hangers in pin trusses.

The steel eye bar is the most commonly used tension member of the solid bar type. This bar is fully described on page 339 of the handbook. The hole in the head depends upon the size of pin which is to be used at the joints of the truss. The dimensions given for the heads are such, that if the bar is tested to destruction, failure will occur in the body of the bar rather than in the head. Adjustable eye bars, such as shown on page 338, may be used for counters; that is, they may be used in truss panels where there is likelihood of a reversal of stress. If the counter stresses are small, iron rods with loops formed by welding, such as shown on pages 346 and 347, may be used. In designing adjustable members, allowance should be made for the reduced section due to the screw threads. In long rods, however, it is advisable to upset the screw end, that is, to make it larger than the body of the bar, so as to give the required area at the root of the thread.

62. Design of Compression Members.—If $l=$length of column in inches and $r=$radius of gyration in inches, then, where the value of $\frac{l}{r}$ in main members does not exceed 100, the *straight line formula* may conveniently be used. An illustrative problem will serve to show the general method to be followed in design.

Illustrative Problem.—Design a 25 ft. channel column for a total load (live, dead, and impact) of 130,000 lb. Lattice bars will connect the channels and prevent them from bending separately. Use the straight line formula

$$\frac{P}{A} = 16{,}000 - 70\frac{l}{r}$$

A trial section should first be determined by assuming $\frac{P}{A} = 12{,}000$ lb. This gives a trial area of $\frac{300{,}000}{12{,}000} = 25.0$ sq. in., which may be furnished by the use of two 15-in. channels at 45 lb. having a total area of 26.48 sq. in. The radius of gyration for one channel about an axis perpendicular to the web is 5.32 in., hence the allowable value of

$$\frac{P}{A} = 16{,}000 - 70\frac{(25)\,(12)}{5.32} = 12{,}050 \text{ lb.}$$

The actual unit stress for this size of channel equals $\frac{300{,}000}{26.48} = 11{,}330$ lb. Thus the column would be well on the safe side and may possibly be decreased in size. Try a 15-in. channel at 40 lb. The allowable value of

$$\frac{P}{A} = 16{,}000 - 70\frac{(52)\,(12)}{5.44} = 12{,}150 \text{ lb.}$$

The actual unit stress would be $\frac{300{,}090}{23.52} = 12{,}800$; hence, these channels are a little too small and the 15-in. 45-lb. channels should be chosen. These should be placed so that the radius of gyration about the axis 2–2, page 227, will equal that about the axis 1–1. This gives the column equal strength in the two directions. For the present the table on page 227 may be used. This table shows that if the second type of column is chosen, E should equal 12.08 in.

Problem 86.—Design a 20 foot channel column for a total load (live, dead, and impact) of 200,000 lb. Use the straight line formula.

ASSIGNMENT 13

CHAPTER XI

USE OF STEEL HANDBOOK

[The use of a steel handbook is necessary for the convenient and rapid design of structural members. By its use, a great many laborious calculations may be avoided and, at the same time, there is much less chance of making error than where each set of computations must be made independently without the convenience of comparing results with other sets. The Cambria handbook is referred to constantly in all the steel courses which follow and the student should become familiar with the use of the tables and with the formulas upon which they are based.]

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63. Rolled Shapes.—The steel used in structures is in the form of single pieces, or combinations of two or more pieces, to which the general term *shapes* is applied. These different structural shapes are made by rolling out rectangular prisms, called *ingots*, that come from the open-hearth furnace. The ingots are sometimes rolled a few times to form what are known as *billets*. The shapes which are mostly used in construction are: square and round rods, flat plates, angles, channels, I-beams. Z-bars, and T-bars. Look in Cambria handbook for illustrations of these sections (pages 2 to 24 inclusive). The student should now review the part of "Strength of Materials" which treats of the properties and manufacture of steel. Also read "Manufacturers' Standard Specifications," pages 353 to 356 inclusive, in the handbook.

The process of rolling I-beams, channels, angles, etc., is in general as follows: The ingots, as they come from the open-hearth furnace, are placed in *soaking pits* below the ground. These soaking pits are kept at a high heat and, when the ingots are white hot, they are taken out and passed several times through the first set of shaping rolls. The rolls for the first

rolling are spread apart nearly the entire depth of the ingot and a piece after passing through this set of rolls is only partly formed and has only the general shape of the finished product. It has such shape, however, that it is easily passed through the next set of rolls and then it is passed to the third or finishing rolls where the final shaping takes place. The pieces, still very hot, are then passed on by movable tables to circular saws where they are cut into required lengths.

The method of increasing sectional areas is shown on page 25 of the handbook. For example, suppose it is desired to roll channels or I-beams having the same depth, but different thicknesses of web. These sections are always rolled horizontally and the increase in thickness of web is accomplished by changing the distance between the rolls, the effect being to change the width of flange as well. Thus, two beams with the same height but different weights differ simply by a rectangle as shown in the handbook illustrations. It will be seen, also, that for an angle with certain size of legs the effect of increasing weight is to change slightly the length of legs, and to increase the thickness.

Angles are the most useful shapes rolled and are made either with equal legs or with unequal legs. They are rolled either out of a rectangular billet—the last pass through the rolls serving to give the exact dimensions—or else from a rectangular plate, the last pass in this case serving to bend the angle into its final shape and giving it the correct dimensions.

64. Tables of Weights.—The weight per cubic foot of the various kinds of timber is given on page 366. This table is useful in computing the dead weight of wooden beams. As before stated, in designing beams, the weight of the beam is first assumed and then the beam is proportioned for this assumed dead weight plus the live load. A more exact value of the dead weight may then be found by using this table and, if necessary, the computations should be revised so that the weight used in the computations will be practically the same as the actual dead weight of the beam.

In all the tables in the Cambria handbook the weight of a cubic foot of steel has been taken as 489.6 lb. Hence, the weight of a piece of steel one foot in length, having a cross-sectional area of one square inch is $\frac{489.6}{144}=3.4$ lb. It follows from this that the weight per linear foot of any piece of steel of uni-

form cross-section may be found by multiplying the area of the cross-section in square inches, by 3.4.

Suppose the we'ght of a 10 in. I-beam is desired, the cross-sectional area being 7.37 (see page 166)

$$7.37 \times 3.4 = 25.06$$

The weight in the handbook is given to the nearest 1/4 lb.

65. Bending Moments and Deflections of Beams.—If the student feels that he has not had sufficient practice in determining maximum bending moments and shears, he should work out the equations given on pages 146 to 149 inclusive. Bending moment diagrams and maximum deflections are also given on these pages.

66. Wooden Beam Tables.—Pages 370 to 375 inclusive, give the safe loads in pounds uniformly distributed for rectangular wooden beams. As stated in the handbook the values given, which include the weight of the beam itself, are for rectangular beams one inch thick with spans from four to forty feet, and depths from four to twenty-four inches. The safe load for a beam of any thickness may be found by multiplying the values given in the tables by the thickness of the beam in inches. The last column of each of the three tables gives a coefficient of deflection, by means of which the deflection for any beam may be obtained, corresponding to the given span and safe load. The deflection is found by dividing the coefficient by the depth of the beam in inches under the given conditions.

In each table the deflection coefficient is given for one species of wood only, as shown, but the deflection for other species may be obtained from these, by proportion, as explained hereafter.

To show the method used in determining the values in these tables, let us consider a white pine beam with a span of 20 ft. The allowable fiber stress (first table) is 700 lb. per square inch. Take the beam formula

$$p = \frac{MX_1}{I}$$

in which

$$M = \frac{1}{8} wL^2(12) = \frac{1}{8} Wl. \qquad l = (20)(12).$$

$$b=1. \quad I=\frac{1}{12}(bd^3).$$

$$X_1=\frac{d}{2},$$

$$p=700.$$

Then substituting these values in our flexure formula,

$$700=\frac{\frac{1}{8}W(20)(12)(\frac{d}{2})}{\frac{1}{12}d^3}$$

$$\text{or } W=3.89\,d^2 \begin{cases}\text{When } d=5 \text{ then } W=97 \text{ (see table).}\\ \text{When } d=10 \text{ then } W=389\text{, etc. (see table).}\end{cases}$$

If the span is 30 ft., then

$$700=\frac{\frac{1}{8}W(30)(12)(\frac{d}{2})}{\frac{1}{12}d^3}$$

or $W=2.59\,d^2$. {when $d=10$ then $w=259$, etc. (see table).

From "Strength of Materials" the deflection at the center of a beam uniformly loaded is

$$D=\frac{5}{384}\cdot\frac{Wl^3}{EI}$$

in which
$$\frac{1}{8}Wl=M=\frac{pI}{X_1}$$

Substituting,

$$D=\frac{5}{48}\cdot\frac{pl^2}{EX_1}$$

The table we are considering is based upon a modulus of elasticity (E) of 1,000,000 lb. per square inch. If the span is 20 ft.

$$D=\frac{5}{48}\cdot\frac{(700)(20)(20)(12)(12)}{(1{,}000{,}000)\ (\frac{d}{2})}$$

$$D=\frac{8.4}{d}\begin{cases}\text{When } d=5 \text{ then } D=1.7=1\dfrac{11}{16} \text{ in. (about).}\\[2ex]\text{When } d=10 \text{ then } D=0.84=\dfrac{13}{16} \text{ in. (about).}\end{cases}$$

The value of 8.4 is as given in the table for a span of 20 ft.

"For the reason that wood has no well-defined limit or modulus of elasticity, the deflections obtained by the use of the coefficients are only approximate and will vary, dependent upon the moisture content of the wood and the character of the loading. The deflections thus obtained are, therefore, useful only as a general indication of the amount of bending to be expected under the given conditions and are not exact as in the case of materials like steel, which has a well-defined elastic limit and modulus of elasticity."

The deflection coefficients in the first table are given for white pine and are based upon a modulus of elasticity of 1,000,000 lb. per square inch (as stated above). The allowable fiber stress in this table is 700 lb. per square inch.

The second table (short-leaf yellow pine) is calculated for an allowable fiber stress of 1000. The deflection coefficients are figured for $E=1{,}200{,}000$.

The third table (white oak and long-leaf yellow pine) is computed for a fiber stress of 1200 and $E=1{,}500{,}000$.

The deflection coefficients for wooden beams, corresponding to different moduli of elasticity than given above, may be easily obtained from the tables. For instance, the deflection coefficients for wooden beams having an allowable fiber stress of 700 with $E=700{,}000$ may be obtained by multiplying the coefficients in the first table by $\dfrac{1{,}000{,}000}{700{,}000}=\dfrac{10}{7}$. In other words, multiply the coefficients given in the tables by $\dfrac{\text{allowable } E \text{ in table}}{\text{new } E}$.

The factor of safety of six (6) is used in these tables. To obtain the safe load or coefficient of deflection corresponding to any other fiber stress multiply the safe load or coefficient from the tables by $\dfrac{\text{new fiber stress}}{\text{allowable stress in table}}$.

The lower dotted line crossing each table indicates a deflection

of $\frac{1}{360}$ of the span for the kind of wood for which the deflection coefficient is given. For spans below the line the safe loads given in the tables will produce a deflection greater than $\frac{1}{360}$ of the span, while those above the line will produce less than this, which is the usual limit of deflection in order to prevent cracking of plastered ceilings. Similarly, the upper dotted line indicates the limit of deflection for the kind of wood for which the coefficient is given, corresponding to a modulus of elasticity of 500,000 for the first table, 600,000 for the second table, and 750,000 for the third table. This line should be used in cases where the deflection must be closely limited.

Table 1.—The upper full zig-zag line in the table gives the limits of the safe loads corresponding to the allowable shearing stress along the neutral axis of the beam. This shearing stress acts along the fibers of the beam at different depths, but is greatest at the neutral axis. It is called longitudinal shearing stress and sometimes becomes a matter of importance, especially in timber,

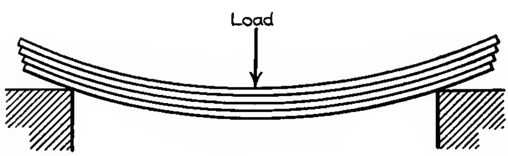

Fig. 109.

where the resistance along the grain is very small. (The existence of horizontal or longitudinal shear can be seen to exist by referring to Fig. 109. The beam shown is subjected to a concentrated load at the center and we know from observation that, if the beam were laminated, the effect would be as shown; that is, the layers would slide on each other as the beam deflects. In a solid beam there is the same tendency to shear the beam longitudinally.) The safe loads above the line, which are based upon the extreme fiber strains, will produce shearing stresses along the axis or with the grain in excess of that allowable, which, in the case of white pine and the other woods of this table is 100 lb. per square inch.

The position of this line which indicates the limit of safe loads

for shearing along the neutral axis was determined by the aid of the following formula:

$$W = \frac{4bdv}{3}$$

in which

W = safe load in pounds uniformly distributed.
d = depth of beam in inches.
b = breadth of beam in inches.
v = allowable shear in the direction of the grain in pounds per square inch.

The above equation is derived from general formula (4) of Art. 60, namely,

$$v = \frac{3}{2}\left(\frac{V}{bd}\right)$$

in which V = total vertical shear at the cross-section in question.

From this general formula for rectangular beams, we can see that the longitudinal shear is a maximum at the same section as the vertical shear. In a simple beam supported at the ends and uniformly loaded, the maximum vertical shear occurs at the supports and equals $\frac{W}{2}$.

Substituting for v, the allowable shearing stress along the grain, and $\frac{W}{2}$ for V, the formula becomes

$$W = \frac{4bdv}{3} \text{ (as previously given).}$$

In this table $b = 1$ and $v = 100$. Substituting,

$$W = \frac{4}{3}(100)\ (d)$$
$$= 133\ d.$$

When $d = 10$ then $W = 1330$ and the line is drawn as shown. See first table.

When $d = 8$, then $W = 1064$, and the corresponding line comes between span 4 ft. and span 5 ft., etc.

Table 2.—The full zig-zag line across the table indicates the limiting spans and loads based on the allowable intensity of shearing stress along the neutral axis of the beam. The values above the full zig-zag line correspond to the shearing stresses

greater than the allowable stress in the direction of the grain for short-leaf yellow pine, while those below the line correspond to shearing stresses less than that allowable, which in this case is assumed to be 100 lb. per square inch.

Table 3.—The lower full zig-zag line indicates the limit of allowable shearing stress along the axis corresponding to the allowable intensity, for yellow pine, of 150 lb. per square inch.

Similarly, the upper full zig-zag line indicates the limits for shearing along the axis for white oak based on an allowable intensity of 200 lb. per square inch.

Care should be taken in designing, to provide sufficient bearing at the points of support so that the allowable intensity of compression across the grain, as given in the table on page 369, is not exceeded. This may be obtained, where necessary, by using bearing plates of harder wood so arranged as to give a large bearing area against the softer beam.

Illustrative Problem.—Required the size of a white pine beam to carry a uniformly distributed load of 2000 lb. per ft. Span 10 ft. Deflection not to be greater than $\frac{1}{360}$ lb. per ft. Allowable fiber stress 700 lb. per sq. in. $E = 1{,}000{,}000$ lb. per sq. in.

$$(2000)\ (10) = 20{,}000 = W.$$

Assume a beam 12 in. wide.

$$\frac{20{,}000}{12} = 1667$$

Looking in the table under a span of 10 ft. a beam 1 in. thick and 15 in. deep will be found to sustain 1750 lb. The lower dotted line is below this load, consequently, the deflection is less than $\frac{1}{360}$ of the span. A 15 in. beam is not an ordinary depth of beam. 16 in. should be selected, (12″x16″). Suppose a beam is required that is not deeper than 14 in.

Let us try a width of 14 in.

$$\frac{20{,}000}{14} = 1430.$$

Looking in the table under span of 10 ft., a beam 14 in. deep is found to safely carry a load of 1524. This is satisfactory and a 14 x 14 in. timber will be selected. There is no danger here of longitudinal shear. The deflection is less than $\frac{1}{360}$ of the span.

The deflection (very nearly) of the selected beam (or beams) under the given loading is

$$\frac{2.10}{14} = 0.15 = \frac{5}{32}\text{in. (about).}$$

Suppose the beam is placed so that it bears upon its support a distance of 6 in. The total bearing area is

$$14 \times 6 = 84 \text{ sq. in.}$$

The allowable crushing strength across the grain (table page 369) is 200 lb. per square inch. The allowable stress is

$$(84)(200) = 16{,}800 \text{ lb.}$$

The actual stress is, allowing 50 per cent. for unequal distribution,

$$\frac{W}{2} = \frac{(20{,}000)(1.5)}{2} = 15{,}000 \text{ lb.}$$

Thus, the beam is safe against crushing at the ends.

A wooden beam that is designed to resist longitudinal shear is always able to resist vertical shear also. This is easily seen to be true when we consider that the actual intensity of horizontal shear is $\frac{3}{2}$ of the intensity of the mean vertical shear (see preceding formula) and that the allowable intensity of stress for horizontal shear is very much lower than for vertical shear (see table page 369).

Illustrative Problem.—Suppose that the allowable fiber stress in a certain kind of timber is specified as 800 lb. per square inch and the modulus of elasticity 1,400,000. It is required to determine the safe load on a 12 × 14 in. beam, span 20 ft. Also, required the deflection resulting from the safe load.

Safe load from first table for a 14 in. beam 1 in. thick with span of 20 ft. is 762.

For beam 12 in. thick, safe load is

$$(762)(12)$$

For an allowable fiber stress of 800 lb. per square inch the safe load is

$$(762)(12)\left(\frac{8}{7}\right) = 10{,}450 \text{ lb.}$$

The safe load per foot is

$$\frac{10{,}450}{20} = 522 \text{ lb.}$$

The deflection resulting from the safe load as given in the table is

$$\frac{8.4}{14}$$

For fiber stress of 800, it becomes

$$\left(\frac{8.4}{14}\right)\left(\frac{8}{7}\right)$$

and for $E = 1{,}400{,}000$ it is

$$\left(\frac{8.4}{14}\right)\left(\frac{8}{7}\right)\left(\frac{10}{14}\right) = 0.49 = \frac{1}{2} \text{ in. (about).}$$

The beam is clearly not in danger of horizontal shear. The bearing area at each end should be sufficient.

Problem 87.—Required the size of a long-leaf yellow pine beam to carry a uniformly distributed load of 1500 lb. per ft. Span 12 ft. Allowable fiber stress 1200 lb. per sq. in. $E = 1{,}200{,}000$. Deflection not to be greater than $\frac{1}{360}$ of the span.

Problem 88.—Determine the safe load on a 12×12 in. long-leaf yellow pine beam. Span 18 ft. Allowable fiber stress 1250 lb. per sq. in. $E = 1{,}500{,}000$. Determine also the deflection resulting from the safe load.

Problem 89.—Determine the safe load on a beam 10x14 in. in cross-section. Span 15 ft. Allowable fiber stress 800 lb. per sq. in. $E = 1{,}100{,}000$. Determine also the deflection resulting from the safe load.

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67. Properties of Sections.—A convenient table is given on pages 152 to 159 inclusive, which gives the "Properties of Various Sections." The properties given include the area of section, distance from neutral axis to extremities of section, the moment of inertia, the section modulus, and the radius of gyration.

It will be remembered that *moment of inertia* was defined in "Strength of Materials," as follows: "The moment of inertia of a plane surface with respect to an axis is the sum of the products obtained by multiplying each elementary area by the square of its distance from that axis." In the beam formula the axis considered is the neutral axis or, in other words, the axis passing through the center of gravity of the cross-section.

From the above definition it can be easily seen that the general term *moment of inertia* refers to *any* axis. For instance, the moment of inertia of a rectangular section can be found about one edge of the rectangle as an axis just as well as it can be found about an axis through the center. Also the moment of inertia of a triangular section can be found about the base of the triangle as an axis just as easily as about an axis through

its center of gravity. It is desirable to remember the values of I given in Fig. 110. The table in Cambria includes these moments of inertia in addition to the moments of inertia of various sections about their neutral axis, as already mentioned. We speak of the neutral axis as if there could be only one neutral axis for a given section. What is meant by the neutral axis for a given section, however, is the axis through the center of gravity of the section at right angles to the direction of the loading.

The moment of inertia of an area about any axis may be computed if we already know, or can find, the moment of inertia about a parallel axis through the center of gravity of the area.

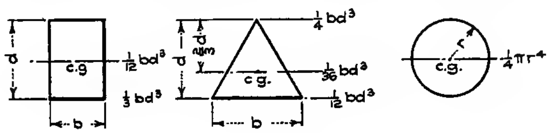

FIG. 110.

The rule may be stated as follows: The moment of inertia of an area with respect to any axis equals the moment of inertia with respect to a parallel axis through the center of gravity, plus the product of the area and the square of the distance between the axes.

Expressed by formula

$$I_1 = I + Av^2 \text{ (see notation, page 144).}$$

The moment of inertia of a rectangle about its base will now be derived.

$$I_1 = \frac{bd^3}{12} + (bd)\left(\frac{d^2}{2}\right)$$

$$= \frac{bd_3}{3}$$

This is the same expression as previously given. Fig. 6, page 152, also shows this same result.

An I-beam section (not considering the small curves in the section) is made up of triangles and rectangles. Fig. 1, page 150, shows an I-beam section as above described. Fig. 111 shows the section divided into triangles and rectangles. The

section is not shown to scale, but is so drawn that the triangles may be more readily seen.

The value of I given in the table of properties of standard and special rolled shapes (pages 166 to 191 inclusive) are not really the values of I for the actual beam, but are instead the exact values for the cross-section which the beam would have if the corners were not rounded. The moment of inertia of the curved areas omitted practically balance the moment of inertia of the curved areas added.

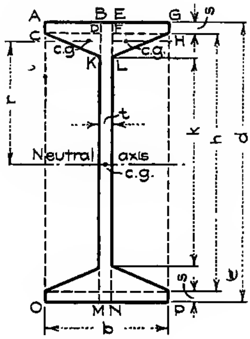

FIG. 111.

It will now be shown how the formulas on pages 150 and 151 were derived. The cross-sectional areas and moments of inertia given in the tables of properties of rolled shapes are computed by means of these formulas. The derivation of the formulas for the I-beam will be taken for illustration. The formulas for the other shapes may be obtained in a similar manner.

Referring to Fig. 111

$$A = AGPO - (CD + FH)h + 2(CDK) + 2(FLH)$$

In an I-beam, $CDK = FHL$, and

$$A = AGPO - (CD + FH)h + 4CDK.$$

$$AGPO = bd$$

$$(CD + FH)h = (b - t)h.$$

$$4CDK = 2(DK)(CD)$$

$$\text{but } 2\,DK = h - k.$$

$$\text{also } CD = \frac{b-t}{2}, \text{ then}$$

$$4CDK = (h-k)\left(\frac{b-t}{2}\right) = \left(\frac{b-t}{6}\right)\left(\frac{b-t}{2}\right) = \left(\frac{b-t^2}{12}\right)$$

(See slope of flange given in Cambria. Slope means amount of rise in a given distance divided by that distance. The slope of flange is 1 in. in 6 in., or $\frac{h-k}{b-t} = \frac{1}{6}$; that is, $h-k = \frac{b-t}{6}$)

Thus, $A \quad = bd - (b-t)h + \frac{(b-t)^2}{12}$

$$= bd - (b-t)(d-2s) + \frac{(b-t)^2}{12}$$

$$= td + 2s\ (b-t) + \frac{(b-t)^2}{12} \text{(same as Cambria).}$$

The beam is ordinarily loaded at right angles to the axis $1-1$. We shall only derive the moment of inertia about this axis. It may be found in a similar manner for the axis $2-2$.

Moment of inertia of $AGPO$ about axis $1-1$, is $\frac{1}{12}bd^3$

Moment of inertia of $(CD+FH)h$ about axis $1-1$, is

$$\frac{1}{12}(b-t)h^3 = \frac{(h-k)h^3}{2} \text{(see slope of flange)}$$

Moment of inertia of CDK about axis through its center of gravity parallel to axis $1-1$, is

$$\frac{1}{36}(CD)\ (DK)^3$$

About the axis $1-1$, it is

$$\frac{1}{36}(CD)\ (DK)^3 + (CD)\left(\frac{DK}{2}\right)r^2 \text{(from formula } I_1 = I + Av^2\text{).}$$

$$\text{but } r = \frac{2}{3}\left(\frac{h-k}{2}\right) + \frac{k}{2} = \frac{2h+k}{6}$$

$$CD = \frac{b-t}{2} = 3(h-k)$$

$$DK = \frac{h-k}{2}$$

The moment of inertia of CDK about axis $1-1$ becomes by substitution

$$\frac{1}{36}(3)(h-k)\frac{(h-k)^3}{8} + (3)(h-k)\left(\frac{h-k}{4}\right)\frac{(2h+k)^2}{36}$$

$$= \frac{(h-k)^4}{96} + \frac{(h-k)^2(2h+k)^2}{48}$$

And I of $4CDK$ about axis $1-1$, is

$$\frac{(h-k)^4}{24} + \frac{(h-k)^2(2h+k)^2}{12}$$

By summation, the moment of inertia of the entire section about axis $1-1$, is

$$\frac{1}{12}bd^3 - \frac{(h-k)h^3}{2} + \frac{(h-k)^4}{24} + \frac{(h-k)^2(2h+k)^2}{12}$$

$$= \frac{1}{12}bd^3 - \frac{1}{8}(h^4 - k^4). \quad \text{(same as Cambria).}$$

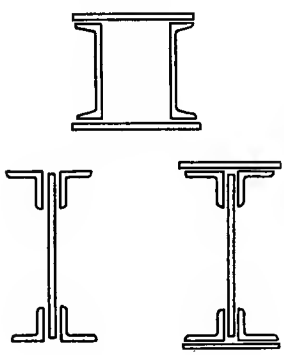

FIG. 112.

The properties of compound shapes such as those shown in Fig. 112 are found in a manner similar to the above. Page 165 gives an extended explanation of the method of finding center of

gravity and moment of inertia of unsymmetrical sections. The table of "Moments of Inertia of Rectangles" pages 192 and 193 saves a great deal of computation in this work. The table explains itself.

Problem 90.—Find the moment of inertia of the section, shown in Fig. 113, about the axis AB.

Problem 91.—(a) Find the moment of inertia of the trapezoid, shown in Fig. 114, about AB. (b) About a horizontal axis through its center of gravity.

Problem 92.—Find the moment of inertia of the I section shown in Fig. 115, about a horizontal axis through its center of gravity. The flanges are equal.

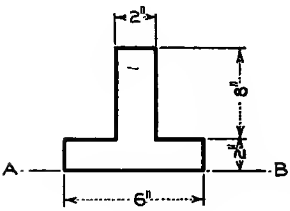

FIG. 113.

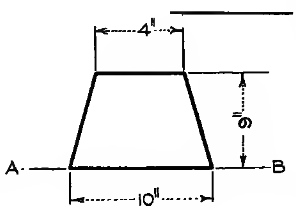

FIG. 114.

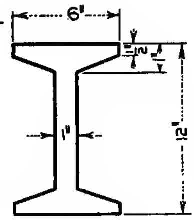

FIG. 115.

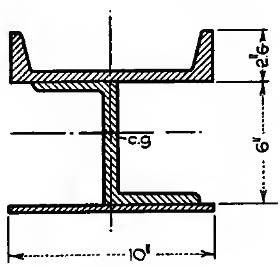

FIG. 116.

Problem 93.—In the column section shown in Fig. 116, the plate is 1/2 in. thick, the area of the 10-in. 15-lb. channel is 4.46 sq. in. (see tables of "Properties of Standard Sections," page 170). Its c.g. is 0.64 in. from the back of the web. Its moment of inertia (axis through its c.g.) parallel to the web is 2.30, and perpendicular to the web is 66.9. The area of the 6-in., 15.6.-lb. Z-bar is 4.59 sq. in. (see "Properties of Z-bars," page 190). Its moment of inertia (axis through its c.g.) parallel to web is 9.11, and perpendicular to web is 25.32. Find the center of gravity of the combined section, also its moment of inertia and radius of gyration about a

pair of rectangular axes parallel and perpendicular respectively to the web of the Z-bar and passing through the center of gravity of the whole section.

Note.—Through the center of gravity of a cross-section there is always a pair of axes about one of which the moment of inertia is a maximum and about the other a minimum. These moments of inertia are called principal moments of inertia and the axes about which they are taken are called principal axes. An axis of symmetry which divides a cross-section symmetrically is always a principal axis. The least radius of gyration and, consequently, the minimum moment of inertia $r=\left(\sqrt{\frac{I}{A}}\right)$ is always used in designing columns. A column bends in a direction at right angles to the axis about which the radius of gyration is a minimum, provided the column is not laterally supported in that direction.

ASSIGNMENT 14

CHAPTER XI—*Continued*

68. Tables for Standard and Special Rolled Shapes.—The properties of the standard shapes manufactured by the different steel companies are practically the same. The *standard* shapes are rolled by all mills, but each steel company also has its own list of *special* shapes. These special shapes, which are different for the different mills, are not as likely to be in stock as the standard shapes.

Standard I-beams are rolled in depths from 3 to 24 in. and standard channels from 3 to 15 in. The different depths of standard I-beams are: 3 to 10 in. consecutively, then 12 in., 15 in., 18 in., 20 in., and 24 in. For channels, 3 to 10 in. consecutively, then 12 in. and 15 in. See properties of standard I-beams and channels, pages 166 to 171 inclusive.

The minimum sizes of the I-beams of different depths are as follows:

3″ – 5.5#	8″ – 18#	18″ – 55#
4″ – 7.5#	9″ – 21#	20″ – 65#
5″ – 9.75#	10″ – 25#	24″ – 80#
6″ – 12.25#	12″ – 31 1/2#	
7″ – 15#	15″ – 42#	

Minimum sizes are more likely to be found in stock. The rolls are made especially for these sections and the heavier sections for a given depth of beam are obtained by spreading the rolls as described in a previous article. A structural designer should know without consulting his handbook at least the minimum sizes of beams and channels rolled.

I-beams, 15 in. and under, take the *base price.* Beams, 20 in. and 24 in. cost a slight amount more per lb. For beams which must be cut to length with a variation not to exceed 3/8 in. more or less than that specified, an additional charge is made.

The standard beams and channels are made to conform to the American Standards, adopted January, 1896, in which the flanges

have a uniform slope of one to six, and the dimensions, proportions, and weights are determined by a regular schedule, as shown on the diagrams on pages 26 and 27. The standard proportions of beams and channels are further shown on page 28. See pages 2 to 8 inclusive, for illustrations of standard and special I-beams. See pages 9 to 11 inclusive, for illustrations of standard and special channels. Also, see tables of weights and dimensions, pages 36 to 39 inclusive.

The principal structural angles now made, are limited in number to conform to the American Standards, adopted December, 1895, and include twelve minimum (called *base*) sizes, or a total of eighty-four sizes for equal leg angles, and nine base, or a total of eighty-six sizes of unequal leg angles, all varying in thickness by 1/16 in., as shown on pages 14 and 16, and tables, pages 40 to 42 inclusive. For special angles, see pages 15, 17, 43, 44, and 45.

The weights of Z-bars and T-bars are those adopted as American Standards in July, 1902. For illustrations, see pages 18 to 24 inclusive. The tables are given on pages 46 and 47.

The drawings of sections, pages 2 to 24 inclusive, are only for the minimum or base sizes of the various shapes. Sections shown on these pages for which more than one weight is stated can be rolled of different thicknesses to produce the stated weights. Others for which only one weight is given cannot be varied. Each section shown is numbered, both in the plates and tables, for convenience in reference and ordering.

The method of increasing the thickness of angles and Z-bars from the minimum has the effect of adding to the length of the legs, as shown on page 25, so that for intermediate and maximum sizes, the legs will be somewhat longer than the minimum or nominal dimensions.

It should be noticed (pages 174 to 187, inclusive) that the areas given for the different angles correspond to the dimensions specified, neglecting the rounded corners which balance each other very nearly. Thus, the area is given exactly only for the minimum sections since we know for all other sections the legs are somewhat longer than for the minimum. However, the areas given in the handbook are slightly on the safe side and should be used in computation.

The student should now study the "Explanations of the Tables of Properties of Standard and Special I-beams, Standard and

Special Channels, Standard and Special Angles with Equal and Unequal Legs, Z-bars and T-bars," given on pages 160 to 162 inclusive. Also, study pages 163 and 164 which give "Examples of Application of the Tables of Properties."

The formula for coefficient of strength is derived as follows for a fiber stress of 16,000 lb. per square inch:

$$M=\frac{1}{8}W \qquad \text{(moment at the center of a beam uniformly loaded)}$$

or
$$W=\frac{8M}{l}=\frac{8pI}{lX_1}=\frac{8pS}{l}$$

but $p=16{,}000$ lb. and $l=12$ in.

$$F=\frac{2}{3}(16{,}000)S. \qquad \text{(See "General formulæ" and "Safe Loads," page 144.)}$$

If any other fiber stress is used, the corresponding coefficient of stress may be found by multiplying F by

$$\frac{\text{new fiber stress}}{16{,}000}.$$

If the new fiber stress should happen to be 12,500 the value can be taken directly from the tables.

The formulas for N and N' are the ordinary formulas for deflection of beams due to uniform and center loads respectively. The student up to this time has been accustomed to seeing the term

$$\frac{wl^2}{76.8EI} \text{ expressed in the form } \frac{5}{384}\cdot\frac{Wl^3}{EI}$$

It is important in proportioning columns to find the least radius of gyration. Angles are often used as small struts or columns. The tables of unequal leg angles on pages 180 to 187 inclusive, give the values of I'', S'', and r'' about the inclined axis $3-3$. This axis is so situated as to give the minimum moment of inertia and consequently the least radius of gyration. The least radius of gyration of equal leg angles comes about the axis $2-2$.

Problem 94.—Determine the proper size of I-beam to support a load of 30,000 lb. at the center of a span of 30 ft., allowable fiber stress 16,000 lb. per sq. in. Also determine the deflection of the beam under the given load.

Problem 95.—Determine the safe load uniformly distributed that can be placed on a 12 in. channel weighing 20 1/2 lb. per ft., the span being 12 ft. and the allowable fiber stress 12,500 lb. per sq. in. The web is to be placed vertically.

When a rolled beam (either channel, I-beam, Z-bar, angle, or T-bar) is required to support one or more irregularly located concentrated loads, the methods used in designing such beams consist in finding the maximum bending moment in inch pounds, and using the beam formula to determine the section modulus; as explained at some length in Art. 60. The beam corresponding to the computed section modulus can then be taken from the tables of properties of rolled shapes.

A table of allowable maximum bending moments in foot pounds, for I-beams and channels, is given on pages 112 and 113 for different depths of beams and fiber stresses 16,000 and 12,500. These bending moments are derived from the beam formula,

$$M=\frac{pI}{X_1}=pS.$$

For an I-beam 3 in. deep and 5.5 lb. per ft. with an allowable fiber stress of 16,000

$$\begin{aligned} M &= (16{,}000)(1.7). \quad \text{(See table of properties page 166.)} \\ &= 27{,}200 \text{ in. lb.} \\ &= \frac{27{,}200}{12} = 2270 \text{ ft. lb. (as given in table.)} \end{aligned}$$

These tables of bending moments may be used for I-beams and channels in order to shorten the computations in their designs. The method used consists in computing the maximum bending moment in foot pounds resulting from the specified loading, the proper section corresponding to a fiber stress of 16,000 or 12,500 lb. per square inch being taken directly from the tables without further computation. To find the maximum allowable bending moment of a certain beam for a fiber stress other than 16,000 or 12,500 multiply the moment given in the table by

$$\frac{\text{new fiber stress}}{\text{fiber stress in table}}$$

To compare an actual maximum bending moment with the table when the allowable fiber stress is not 16,000 or 12,500, multiply the actual moment by

$$\frac{\text{fiber stress in table}}{\text{new fiber stress}}$$

In some cases two or more beams may be bolted together side by side to form a girder, in which case cast iron separators with bolts should be used to hold the various members together. Separators should be placed at each end of the girder, at points of concentrated loading, and for uniform loading should be located at distances apart not greater than twenty times the width of the smallest beam flange, in order to laterally support the upper flanges which are in compression and prevent their failure by buckling. The separators should fit closely between the beam flanges so as to unite the beams forming the girder and thereby cause them to act together in resisting the load. A table of separators is given on page 50.

When separators are not used and a beam is not supported laterally, the fiber stress used in designing should be lower than the allowable stress previously given, if the beam is longer than twenty times the flange width. This is done to prevent undue strains in the compression flange considered as a column. Columns 3 and 6, page 71, give the reduction, in values of the allowable fiber stress, to use for the different ratios of span length to flange width. Do not consider the heading of the columns mentioned above for the present.

It follows from the above that a flange should be supported laterally at distances not exceeding twenty times the flange width. If the lateral supports are farther apart than this the table referred to above will give the proportion of the allowable stress to use.

Illustrative Problem.—Required the depth and weight of an I-beam to span a 30 ft. opening with no lateral supports. Maximum bending moment is 80,000 ft. lb. Allowable stress is 16,000.

Let us try a special 15-in. 60-lb. special I-beam. This I-beam has a bending moment of 108,270 ft. lb. when $f=16{,}000$ (page 112), and its flange width is 6 in. (page 168).

$$\frac{l}{b}=\frac{(30)(12)}{6}=60$$

Proportion of allowable fiber stress to use $=0.51$ (page 71). Thus, the maximum bending moment which this beam will sustain is

$$108{,}270(0.51)=55{,}218 \text{ ft. lb.}$$

Referring to page 112 it is seen that this I-beam is not of sufficient strength and a 20-in., 65-lb. I-beam will be selected. The width of flange for this I-beam is 6.25.

$$\frac{l}{b}=\frac{(30)(12)}{6.25}=57.6$$

Proportion of allowable fiber stress to use $=0.54$ (page 71).

$$(156{,}000)\ (0.54) = 84{,}240 \text{ ft. lb.}$$

Thus the 20-in. 65-lb. I-beam should be used. If a beam were required not deeper than 20 in., a heavy weight special I-beam would be found to serve the purpose.

Problem 96.—A channel span is to be 12 ft. with no lateral supports, and the maximum bending moment is 60,000 ft. lb. Allowable stress 16,000. Determine the depth and weight of channel to be used.

The tables of safe loads for I-beams, channels, angles, T-bars, and Z-bars are given on the following pages:

I-beams, pages 84 to 94 inclusive.
Channels, pages 95 to 100 inclusive.
Angles, pages 114 to 140 inclusive.
T-bars, page 141.
Z-bars, pages 142 and 143.

The safe loads are in pounds uniformly distributed for all usual spans based upon extreme fiber stresses of 16,000 lb. per square inch. These loads include the weight of the steel shape itself, which should be deducted in order to obtain the external load that it will safely carry.

The safe loads given in the tables are derived from the beam formula in exactly the same manner as the safe loads for wooden beams.

In cases where intermediate lateral support is not provided, the safe loads shown in the tables must be reduced, and for beam and channels the amount of this reduction can be determined by reference to the table on page 71. The method of reducing safe load is the same as the method of reducing allowable fiber stress or maximum bending moment already explained.

In some instances the allowable deflection will govern the design rather than the transverse strength, as in the case of beams carrying plastered ceilings, in which the deflection should be limited to 1/30 in. per foot of span, or 1/360 of the distance between supports in order to avoid cracking the plaster.

This limit of deflection is indicated in the tables by full horizontal lines, the figures below which correspond to loads or spacings for the given spans that will produce greater deflections than the allowable limit for plastered ceilings.

The deflection limits of the tables of safe loads have been calculated for the total loads, including the weight of the section used as a beam. The superimposed live load will not produce all of this deflection, and therefore the deflection limit of the tables includes an element of safety for the reason that the beams will be deflected, after being put in place, by their own weight before the plastering is applied.

In cases where the deflection limits the use of the beam for the safe loads corresponding to the fiber stresses of the tables, the beam may be used with a less load such as to produce only the allowable deflection. The lesser load corresponding to the limit of deflection may be obtained for any span from the table of safe loads by following this rule:

Rule.—Multiply the safe load above the heavy line of the tables by the square of the corresponding span in feet and divide the product by the square of the required span. The result will be the required load corresponding to the limit of allowable deflection for plastered ceilings. Under "Examples of Application of Tables of Safe Loads and Tables of Spacing," study examples 1 and 2, pages 81 and 82.

Problem 97.—What is the proper size of an I-beam to carry a superimposed load of 40,000 lb. uniformly distributed? Span is 20 ft. Allowable fiber stress 16,000 lb. per sq. in. The deflection is to be such as not to crack a plastered ceiling. Beam is laterally supported.

Problem 98.—Find the safe load for a 12-in. 30-lb. channel for a span of 30 ft. Deflection is not to be greater than 1/30 in. per foot of span. Beam is *not* supported laterally.

A table of coefficients of deflections for rolled shapes used as beams, subjected to their safe loads uniformly distributed, and accompanying explanations with examples, are given on pages 76 and 77.

Problem 99.—Find the deflection of a 15-in. 42-lb. I-beam, for a span of 25 ft., and a maximum fiber stress of 16,000 lb. per sq. in. under its safe load uniformly distributed.

Problem 100.—Find the deflection of a 3 1/2″×2″×1/2″ angle, supported at the ends on its long leg as a horizontal base, for a span of 10 ft., and a maximum fiber stress of 16,000 lb. per sq. in., under its safe load uniformly distributed including its own weight.

I-beams and channels, when used as beams for very short spans in which the ratio of length of span to depth of beam is small, should be examined for safe strength of the web considered as a column subjected to crippling due to the shearing strains. The method of computing minimum safe spans for maximum safe loads (as regards transverse strength) based upon the crippling of the web is considered in Art. 70. It will be sufficient for the present for the student to know that I-beams and channels subjected to safe loads, as regards transverse strength, can be so short that crippling of the web is likely to result. The minimum spans for I-beams and channels and the corresponding safe loads are given on pages 74 and 75.

Problem 101.—Find the minimum span to support a maximum safe load on a 24-in. 80-lb. I-beam.

A table of the spacing of I-beams for a uniform load of 100 lb. per square foot and maximum fiber stress of 16,000 lb. per square inch, is given on pages 101 to 111 inclusive. This table is convenient for the design of cross beams in floors. The spacing for other intensities of loading may be obtained as explained at the bottom of each page of this table.

Problem 102.—Find the spacing of 20-in. 80-lb. I-beams with span of 30 ft. to carry a load of 150 lb. per sq. ft.

ASSIGNMENT 15

CHAPTER XI—*Continued*

69. Column Tables.—The load which a column will bear before failure depends upon the ratio of its unsupported length to the least radius of gyration of its cross-section. This ratio, expressed as $\frac{l}{r}$, occurs in all column formulas since it is the most important factor as regards strength. The radius of gyration of circular or rectangular sections may conveniently be written in terms of the least outside dimension, and formulas for columns of these shapes often have the radius of gyration so expressed.

It is also true, that the condition of the ends plays an important part in the ultimate load which a column will support. For example, if the ends are flat, the column cannot bend sideways as readily as though they rested on steel pins and were free to turn in either direction on the pin. Also, if one end of the column is flat and the other end bearing on a pin, the tendency to fail by lateral bending would be less than would be the case if both ends were pin bearing, and more than would be the case if both ends were flat.

In common practice, therefore, there are three conditions of the ends of columns which affect their strength.

1. Both ends flat or square.
2. One end pin bearing and the other end flat or square.
3. Both ends pin bearing.

There are two classes of formulas by which columns are figured: (1) formulas on a rational basis with experimental constants; (2) formulas wholly empirical.

A formula of the first class, which was of most general use until of late years, and which is still used to a considerable extent, is known as "Gordon's Formula," so-called from the name of the author. It is known to rest upon an imperfect theoretical basis, but with the insertion of experimental constants it has given safe results. Its general form is:

$$P = \frac{f}{1 + \frac{l^2}{cr^2}} \qquad (1)$$

in which

P = ultimate strength in pounds per square inch of cross-section of the column.

f = ultimate strength of the material in short prisms—pounds per square inch.

c = a constant depending on the material considered and determined experimentally by tests of actual columns.

l = length of column in inches.

r = least radius of gyration of the column in inches.

The general formula given above is without reference to the arrangement of the ends of the column. Assume that c is a constant for ends entirely unrestrained against turning, called round ends, Fig. 117. Now, if both ends of a column are rigidly

FIG. 117.

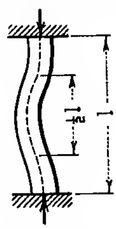

FIG. 118.

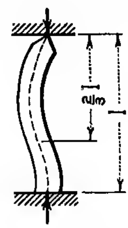

FIG. 119.

fixed, as in Fig. 118, the load which such a fixed column will carry before failure is the same as may be carried by a round ended column of one-half its length. Thus, $\frac{l}{2}$ must be substituted for l and the formula becomes for both ends fixed:

$$P = \frac{f}{1 + \frac{l^2}{4cr^2}} \tag{2}$$

If one end of the column is fixed and the other end free to turn, as in Fig. 119, $\frac{2}{3}l$ must be substituted for l, and the formula becomes

$$P = \frac{f}{1 + \frac{4l^2}{9cr^2}} \tag{3}$$

In practice, however, no column is perfectly free to turn at the ends, and it is seldom the case that a column can be considered absolutely fixed; thus, it becomes necessary to assume arbitrary

values for c in equation (1) for the three cases, instead of using the modified formulas (2) and (3).

In the tables in the handbook on "Strength of Steel Columns or Struts," pages 200 to 203 inclusive, the value of f for soft steel is taken at 45,000 and for medium steel at 50,000. Also, for pin bearing, c is taken at 18,000; for square bearing 36,000; and for pin and square bearing 24,000. These values have been adopted from the results of tests.

Recent tests of full size columns give somewhat unfavorable results for those having a comparatively low ratio of $\frac{l}{r}$. In view of this fact, many engineers advocate the use of one formula only, both for columns with pin ends and for those with square ends, assuming that in the latter case the accidental eccentricity of loading due to a possible unevenness of bearing and other conditions are liable to induce bending moments fully as great as would occur in columns with pin ends.

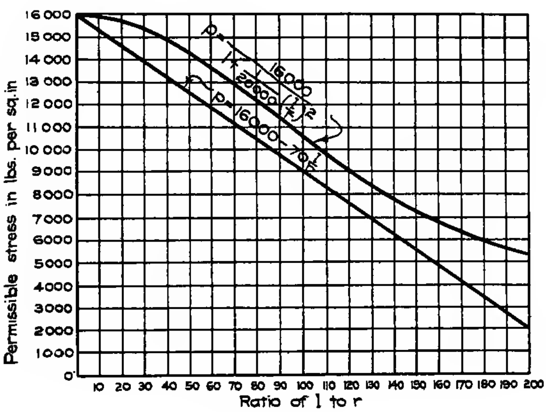

FIG. 120.

Gordon's formula, as given above, can be made a safe load formula by substituting the allowable compression per square inch in a short prism for the value f. The formula in most general use for safe loads on medium steel columns is

$$p = \frac{16{,}000}{1 + \frac{l^2}{20{,}000 r^2}} \tag{4}$$

where p is the average allowable stress per square inch on the

column. The safe load in pounds is then found by multiplying p by the area of the cross-section of the column in square inches.

The curve obtained by plotting different values obtained from equation (4) is shown in Fig. 120; also the straight line representing the equation

$$p = 16{,}000 - 70\,\frac{l}{r} \qquad (5)$$

which is the column formula for medium steel adopted by the American Railway Engineering and Maintenance of Way Association. This latter formula (called the *straight line* formula) is purely empirical and thus comes under the second class of column formulas previously mentioned.

Formulas (4) and (5) are to be used for main members and for a value of $\frac{l}{r}$ not to exceed 100. For secondary members, such as lateral struts, a higher unit stress may be used in place of the 16,000 lb. per square inch and $\frac{l}{r}$ may be as high as 125.

An empirical formula for wooden columns adopted by the U. S. Department of Agriculture, Division of Forestry, is given on pages 376 and 377 of the handbook. The tables given on these pages are based on this formula.

Study carefully pages 204 and 205 in the handbook and familiarize yourself with the use of all the tables from page 195 to 290 inclusive.

Problem 103.—Find the safe load on a Z-bar column, square ends, made up of 4 Z-bars 6 in. deep × 5/8 in. thick and 1 web plate 7 3/4 in. × 5/8 in., 30 ft. long. See tables on pages 252 and 253 of Cambria.

Problem 104.—What is the least radius of gyration of a 2″×1 1/2″×3/8″ angle?

Problem 105.—What is the radius of gyration of two angles 6″×6″×5/8″ riveted together, about an axis parallel to the legs which are joined? Use tables on pages 176 and 177 and compare with the result given in table on page 195.

Problem 106.—What is the radius of gyration, about an axis parallel to the longer legs, of two angles 5″×3 1/2″×5/8″, riveted together; the short legs being joined. Use tables on pages 182 and 183 and compare with the result given in table on page 199.

Problem 107.—Find the least radius of gyration of two angles 4″×3″×5/16″ riveted together, if held apart 3/8 in.; the longer legs being joined. Compare result with table on page 197.

Problem 108.—Find the safe load on the strut of Problem 103, using tables for square bearing, medium steel, pages 202 and 203. Use factor of safety 4. Length of strut = 7.6 ft.

Problem 109.—Suppose that the strut of Problem 109 is held against bending in one direction at the center of its length, by a member riveted to the short legs of the angles. Find safe load for same conditions as in Problem 109, with the above exception.

Problem 110.—What is the safe load on a round cast iron column 8 in. diameter and 3/4 in. shell, the length being 18 ft? Factor of safety = 6. Square ends.

Problem 111.—What is the safe load on a round cast iron column 8 in. diameter and 1 1/4 in. shell, the length being 22 ft. and factor of safety of 8? Square ends.

Problem 112.—Find the safe load on a column 30 ft. long, made of two channels 9-in. 15-lb. latticed. What will be the safe load if two side plates are added, 11 in. × 3/8 in.? Column is of medium steel with square ends.

The most economical sections for columns are those which have the material so distributed as to give the greatest value for the radius of gyration. For instance, Fig. 121 represents a column composed of four angles riveted together as shown. Fig. 122 shows a column also made up of four angles, but instead of being riveted together, they are connected by lattice bars. It is a

Fig. 121.

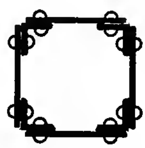

Fig. 122.

self-evident fact that, if the four angles in each of these types of columns are of the same size, the section of the column shown in Fig. 122 will have the larger value of radius of gyration and, if the columns were of the same height, the column of Fig. 122 would carry the greater load.

Problem 113.—Refer to the types of columns shown in Figs. 123 and 124. Which will be the stronger for a column 25 ft. long if the channels in each column are 12-in. 20.5-lb., and the I-beam in each case is 8-in.18-lb.? What will each column safely carry if the length is 15 ft. 6 in., and the factor of safety 5? Consider columns of medium steel with square ends.

In a preceding assignment, the statement was made, that the allowable fiber stress in beams supported laterally at distances

exceeding twenty times the flange width, should be reduced according to the amount specified in the table on page 71. This table is based on the safe load column formula given on page 70. The reason for the formula in this shape will now be explained.

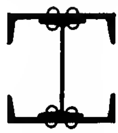

Fig. 123.

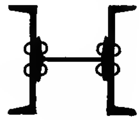

Fig. 124.

The top flange of an I-beam or channel in a beam subjected to bending is stronger in compression than a column standing alone without any connection with the web. In other words, the flange in compression, due to bending, is strengthened as regards bending side-ways and creating stresses similar to those produced in a column, by being rigidly connected with the web. Gordon's ultimate load formula for medium steel and fixed ends is

$$p=\frac{50{,}000}{1+\dfrac{l^2}{36{,}000\, r^2}}$$

Allowing a small factor of safety on account of the extra strength mentioned above, the resulting safe load may be expressed as follows:

$$p=\frac{18{,}000}{1+\dfrac{l^2}{36{,}000\, r^2}}$$

Assuming the flange a rectangle with axis through center of rectangle parallel to web of beam,

$$r^2=\frac{I}{A}=\frac{1}{12}b^2$$

in which b = width of flange in inches.

Substituting,

$$p=\frac{18{,}000}{1+\dfrac{l^2}{36{,}000\, b^2}}$$

as given in handbook.

The student should now study carefully pages 70 and 71.

Problem 114.—The allowable stress to be used in the design of a certain I-beam span is 14,000 lb. per sq. in. A 15-in. 42-lb. I-beam is selected and the beam is 25 ft. long without lateral supports. Required the allowable unit stress for direct flexure in extreme fiber.

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70. Crippling of Web of I-beams and Channels.—The statement was made previously in this course, that loads on I-beams and channels can be so great that crippling of the web will result, and in this connection the tables on pages 74 and 75 of the handbook were employed. The student should now study the reason for this crippling given on pages 72 and 73.

An attempt will now be made to show the action of internal stresses in beams on the crippling of the web.

The general formula for the intensity of horizontal shear at a given point of any form of cross-section is as follows:

$$v=\frac{VQ}{Ib'}$$

in which

V = total shear at the cross-section in question.

Q = Statical moment about the neutral axis of that portion of the cross-section lying either above or below (depending upon whether the point in question is above or below the neutral axis) an axis drawn through the point in question parallel to the neutral axis.

I = moment of inertia of the section about the neutral axis.

b' = width of beam at the given point.

For rectangular beams the only change in the general formula is that b' becomes a constant and equal to b.

The case of horizontal shear is shown in Fig. 125. Let the fiber stresses at section m be represented by p and those at section n by p_1, while the longitudinal variations of the fiber stresses between these two sections are indicated, at section n, by the cross-shaded areas. This increase of horizontal stress from one section to another (which we know to be true since the bending moment M increases from the ends toward the center of span and with it the intensity of the horizontal stresses) induces a force at every longitudinal layer tending to slide it past the next section

above it; and this sliding or shearing force, which increases at every layer, attains its maximum intensity at the neutral axis.

In addition to the longitudinal shear at any point, as explained above, there co-exists a vertical shear and the intensity of this vertical shear is equal to the intensity of the horizontal shear. This may be proved as follows:

Fig. 126 represents an infinitely small portion of the side of a beam at any given point. The sides of the element will be repre-

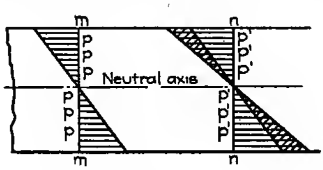


Fig. 125.

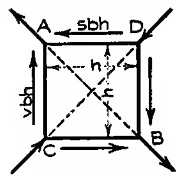


Fig. 126.

sented by h, and the breadth of the beam at this point by b. Now there are two sets of shearing forces acting upon it, one vertical and the other horizontal; and these shears form two pairs of couples, acting as indicated by the vertical and horizontal arrows. For an infinitesimal distance h the horizontal fiber stresses balance each other and need not be considered. If the intensity of the horizontal shear at this point be represented by s, and that of the vertical shear by v, then in order that the total horizontal and vertical shears acting on the partical may be in equilibrium, the moments of these two shears must be equal, thus:

$$(sbh)h = (vbh)h$$
$$\therefore\ s = v.$$

Since the intensities of the horizontal and vertical shears are equal, they will both be represented by the common symbol v.

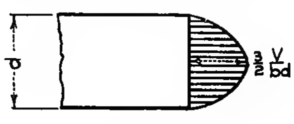


Fig. 127.

Using our general formula it should be clear that the intensity of the shear at the top and bottom of a beam is zero and, by substituting the proper values of the separate terms for a rectangular cross-section, the student will find that the intensity of shear (horizontal and vertical) along a vertical cross-section for a

rectangular beam varies as the ordinates to a parabola, as shown graphically in Fig. 127.

The general formula applies to any shape of cross-section provided b' is considered the breadth of beam at the point in question. Fig. 128 shows the variation of the intensities of shear for a 24-in. I-beam at 80 lb. per ft. with an assumed vertical shear on the section of 72,000 lb. Now the area of the web (which is

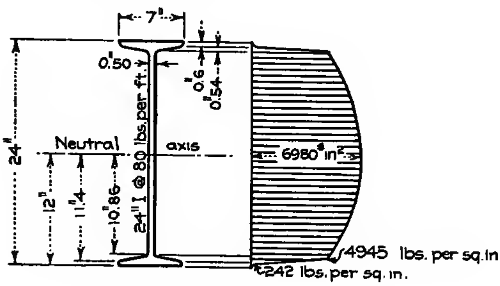


FIG. 128.

21.72 in. deep and 0.50 in. thick) = 10.86 sq. in.; and, assuming that it takes all the shear, the average intensity of this stress equals

$$\frac{72,000}{10.86} = 6630 \text{ lb. per sq. in.}$$

which is only 5 per cent. less than the maximum. About the same per cent. of difference will be found between the maximum and the average shears using other weights of I-beams and it is therefore, customary to assume that the shear is distributed evenly over the web.

We have shown that shear on a vertical cross-section is greatest at the neutral axis and at this axis the horizontal fiber stress is zero; in other words, there are no other forces on the web but those of shear. Consider again the infinitely small prism shown in Fig. 126, but this time assume it to lie at the neutral axis. The shearing forces acting on this prism will develop inclined stresses of tension and compression. The value of these inclined stresss which are indicated by the diagonal arrows, Fig. 126, may be found by resolving the shearing forces into components parallel to AB and CD. The value of each component is found to be $\frac{vbh}{\sqrt{2}}$. The effect of these components is to produce on CD a total

tension of $\frac{2vbh}{\sqrt{2}}$ and on AB a total compression of the same amount. Since the length of the diagonals is $h\sqrt{2}$, the intensities of these inclined stresses are equal and with a magnitude of $\frac{2vbh}{(\sqrt{2})(h\sqrt{2})(b)}=v$. It therefore follows that at the neutral axis there exists a tension and compression at angles of 45° to that axis and that the intensity of these forces is equal to that of the shear.

Above and below the neutral axis the direction and magnitude of the inclined stresses are not as above found, due to the fact that the final value of the tension or compression at any point would have to be obtained by combining the horizontal fiber stresses due to bending with the inclined stresses due to shear. At the end of a beam, however, where the shear is a maximum and the bending moment a minimum, the inclined stresses act approximately at 45° with the neutral axis throughout the entire depth of beam and this is where sidewise buckling of the web may be looked for.

The web at the end of beam is, therefore, subjected to a compression along a line at an angle of 45° with the horizontal, this compression having the same intensity as the shear, and this intensity (for I-beams, channels, and plate girders) is $\frac{V}{\text{area of web}}$ since we have just seen that the shear v is distributed almost uniformly over the web. Take 1 in. in width of the web, the thickness being t, and length l, and apply the column formula (page 202) for medium steel, square bearing, and safe unit stress.

$$\text{Then, } \frac{V}{A}=\frac{12{,}000}{1+\dfrac{l^2}{36{,}000\left(\dfrac{1}{12t^2}\right)}}=p$$

$$\text{or } p=\frac{12{,}000}{1+\dfrac{l^2}{3000t^2}}$$

Let $c=$ depth of web in clear. Then $l=c\sqrt{2}$, since l is the direction in the clear at an inclination of 45° with the neutral axis, or

$$l^2 = 2c^2$$

$$\text{and } p = \frac{12{,}000}{1 + \dfrac{c^2}{1500t^2}} \quad \text{(see page 73)}$$

in which $p =$ average intensity of vertical shear in pounds per square inch.

Problem 115.—Find the maximum safe load uniformly distributed for a 20-in. 65-lb. I-beam and the corresponding minimum safe span based upon crippling of the web. Compare results with the values given in table on page 74.

---

71. Rivet Tables.—Rolled shapes are connected to one another and to other shapes by means of rivets. Rivet holes are generally punched 1/16 in. larger than the rivet to be used; but, in more particular work, they are punched 1/8 in. smaller and, after assembling the parts to be riveted, are reamed to 1/16 in. larger than the rivet. Holes in metal of greater thickness than the diameter of the rivets are usually drilled, as they are difficult to punch and because the punching of thick metal is considered injurious to it.

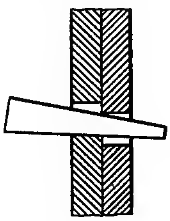


FIG. 129.

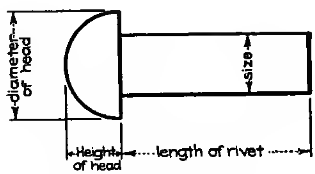


FIG. 130.

The sizes of rivets used in structural steel work vary from 3/8 to 1 1/8 in. in diameter. Those in most general use are 3/4 and 7/8 in. The smaller ones are used only in very light members to avoid cutting out too much of the sectional area. Rivets larger than 7/8 in. are difficult to drive, as they are used only in cases where it is impossible to get in enough of a smaller size, owing to lack of available space.

With careful men to lay out and punch the work, the parts go together without much difficulty, but to bring two holes in line which are slightly off center, a *drift pin* may sometimes be needed as shown in Fig. 129. In riveting up work, a heater prepares the

rivet to proper heat, then gives it to a helper who puts it in place, where it is held by the riveter for an instant while the helper places a *dolly* bar against its head. The riveter then forms the other head, but care must be exercised in finishing that it is not struck too cold, for fear that the head will fly off.

Rivets that are driven at the manufacturers are called *shop* rivets and those driven where the structure is erected are called *field* rivets. Most of the riveting in the shop is done with a machine and many of the field rivets are also driven by the same method.

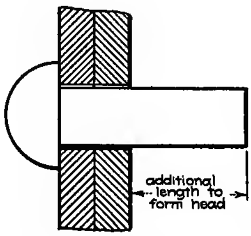


FIG. 131.

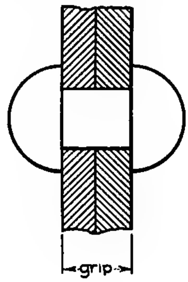


FIG. 132.

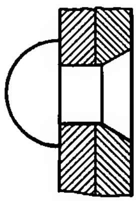


FIG. 133a.

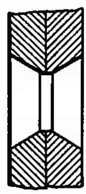


FIG. 133b.

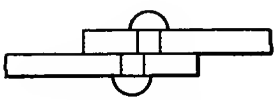
Each rivet before driving has one head as represented in Fig. 130. Fig. 131 shows the rivet inserted in the hole and ready for driving. Fig. 132 shows the rivet completely driven with the other head formed. This kind of rivet is called a *button head* rivet.

In some cases it is objectionable to have a rivet head protrude beyond the surface of the metal. In order to avoid this, one or both ends of the hole may be enlarged to a conical form, and a shorter rivet used. The rivet should be long enough to fill the enlarged hole. It is very difficult to get a rivet that will just fill the hole, and the head usually projects about 1/8 in. beyond the surface. This style of rivet is called a *counter-sunk* rivet. See Fig. 133. If it is objectionable to have a countersunk head project even 1/8 in. beyond the metal surface, it should be chipped off with a chisel.

In case button-head rivet heads are too high and it is not desirable to use countersunk rivets, the heads may be flattened or hammered, when hot, to a height of 3/8, 1/4 or 1/8 in.

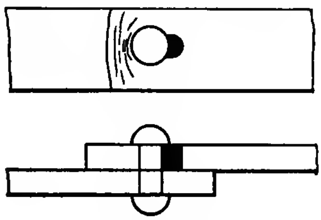
A table on page 342, Cambria, gives the dimensions of button and countersunk heads for rivets from 1/4 to 1 1/2 inches in diameter.

The amount of metal held by the rivet is called the *grip*. It has been found that there is a practical limit to the length or grip a rivet may have, which if exceeded, renders it almost impossible to drive tight rivets. Those who have investigated this subject now specify that the maximum grip of a rivet shall never exceed four times the diameter of the rivet. On page 343 is given a table giving "Length of Rivets Required for Various Grips Including Amount Necessary to Form One Head."



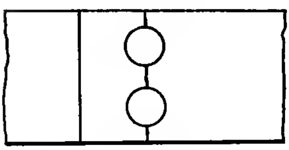
SHEAR

Fig. 134a.



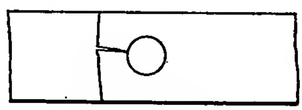
BEARING

Fig. 134b.



TEARING

Fig. 134c.



BREAKING THROUGH

Fig. 134d.

Page 315 shows the conventional signs used in preparing drawings to indicate where the rivet is to be driven and what type of head is desired.

The several ways a riveted connection may fail are shown in Figs. 134a, 134b, 134c, and 134d.

a. By shearing off the rivet.

b. By crushing fiber of thinnest plate, namely, bearing.

c. By tearing plate between rivet holes.

d. By rivet breaking through plate.

The joint *b* may fail by crushing the fiber of rivet, but, since the bearing value of the rivet must be determined by the thickness of

the thinnest plate, the crushing of the plate controls. Fig. 135 shows rivets which have shear at two sections and they are said to be in double shear.

There are rules for rivet spacing in structural work whereby the c and d ways of failure are prevented. These rules accomplish this by stating the allowable spacing of rivets, and the allowable distance from edge of riveted piece to the center of rivet hole, under the different conditions. General rules for rivet spacing are given on page 322 of the handbook, and a table of same at the bottom of page 321.

The shearing value of a rivet is equal to the area of its cross-section in square inches, multiplied by the permissible shear per square inch. Thus, the shearing value of a 3/4 in. rivet at 12,000 lb. per square inch $= 0.4418 \times 12{,}000 = 5300$ lb. The bearing value of a rivet is equal to the diameter of the rivet multiplied by the thickness of metal on which it bears (both in inches) multiplied by the permissible bearing per square inch. Thus, the bearing value of a 3/4 in. rivet on a 3/8 in. plate, at 24,000 lb. per square inch $= 3/4 \times 3/8 \times 24{,}000 = 6750$ lb. Rivets in double shear would have to be sheared in two planes before the joint could fail and they would thus have twice the shearing values of rivets in single shear. In most cases, however, when rivets are in double shear, their bearing value determines the strength of the joint.

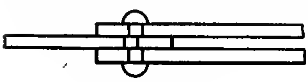
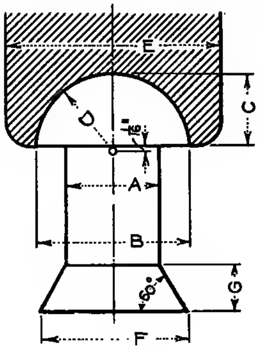


FIG. 135.

In Fig. 135 suppose the plates are all 3/8 in. in thickness, with 3/4 in. rivets. Each rivet then (with unit values assumed above) is good for $5300 \times 2 = 10{,}600$ lb. in shear and 6750 lb. in bearing; and thus the bearing values govern. If the center member were 1/2 in. thick the bearing value of the rivets would then be $3/4 \times 1/2 \times 24{,}000 = 9000$ lb. each, which is still less than their shearing value. If this member were 5/8 in. thick, the bearing value would be $3/4 \times 5/8 \times 24{,}000 = 11{,}260$ lb.; therefore, the double shearing value of the rivets would determine the strength of the joint. In designing a riveted connection, great care must be taken always to use the least value a rivet can have under the circumstances, whether in single shear, double shear, or bearing.

Shop rivets are generally calculated at higher values than field-driven rivets, it being assumed that they are driven by machine riveters capable of exerting a heavy pressure. Many

firms have designed rivet dies for their machines and the sizes of a well-known firm are shown in Fig. 136. It is important to avoid placing rivets in positions that cannot be reached by the machine riveter. Rivets that cannot be reached by the machine



STANDARD RIVETS AND DIES

A	B	C	D	E	F	G
$\frac{3}{8}$	$\frac{11}{16}$	$\frac{9}{32}$	$\frac{11}{32}$	1	$\frac{5}{8}$	$\frac{9}{16}$
$\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$1\frac{1}{4}$	$\frac{13}{16}$	$\frac{1}{4}$
$\frac{5}{8}$	$1\frac{1}{16}$	$\frac{15}{32}$	$\frac{17}{32}$	$1\frac{1}{2}$	1	$\frac{5}{16}$
$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{9}{16}$	$\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{3}{16}$	$\frac{3}{8}$
$\frac{7}{8}$	$1\frac{7}{16}$	$\frac{21}{32}$	$\frac{23}{32}$	2	$1\frac{3}{8}$	$\frac{7}{16}$
1	$1\frac{5}{8}$	$\frac{3}{4}$	$\frac{13}{16}$	$2\frac{1}{4}$	$1\frac{9}{16}$	$\frac{1}{2}$
$1\frac{1}{4}$	2	$\frac{13}{16}$	1	3	$1\frac{15}{16}$	$\frac{5}{8}$

FIG. 136.

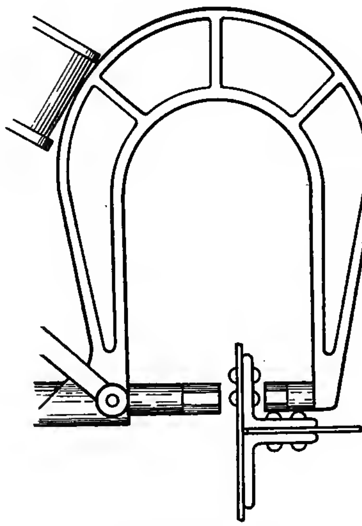


FIG. 137.

must be driven by hand at a much greater cost. A typical machine riveter is shown in Fig. 137.

Tables, pages 316 and 317, give the shearing and bearing values or rivets, computed for four different permissible shears, and for four different permissible values for bearing. All bearing values between the lower and upper zigzag black lines are greater than single and less than double shear for the corresponding dimen-

sions so that in case of single shear, the single shearing value governs, and in case of double shear, the bearing value governs the design.

The student should now familiarize himself with the tables for standard connection angles for I-beams and channels, pages 48 and 49. The different connection angles are shown on page 51. Also, note the manner of connecting beams of the same or different sizes, framing opposite, bottoms or tops flush, pages 52 and 53.

Look over the following tables carefully. They should be easily understood by the student. "Standard Spacing of Rivet and Bolt Holes through Flanges and Connection Angles of I-beams and Channels," pages 56 and 57. "Areas of Rivet Holes," pages 320 and 321—"Maximum Size of Rivets in Beams, Channels, and Angles," page 320.

Problem 116.—Two bars 3/8 in. and 1/2 in. in thickness respectively are connected by two 3/4 in. rivets. What is the value of the riveted joint or, in other words, what total stress will it carry? Shearing value, 7500 lb. per sq. in. Bearing value, 15,000 lb. per sq. in. Use table.

Problem 117.—If in Problem 116 both bars had been 1/2 in., what would have been the value of the riveted joint?

Problem 118.—A joint consists of two 3/8 in. bars extending in one direction and one 1/2 in. extending in the opposite direction, the bars being connected together with two 7/8 in. rivets. What would be the value of the riveted joint assuming 12,000 lb. for shear and 24,000 lb. for bearing?

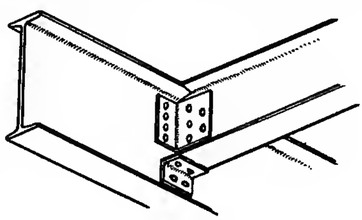


Fig. 138.

Problem 119.—Using standard connections, shown on page 51 of the handbook, with allowable bearing of 20,000 lb. per sq. in. and allowable shearing value of 10,000 lb. per sq. in., what would be the allowable end reaction (see Fig. 138) of

(a) A 12″−31 1/2 lb. I framed into a 15″−42 lb. I.
(b) A 15″−42 lb. I framed into a 18″−55 lb. I.
(c) A 15″−42 lb. I framed into a 20″−65 lb. I.

(d) If a bracket angle, 4″×3″×3/8″, with two 3/4 in. rivets through web of 20-in. I be placed under the 15-in. I, what would be the increase in allowable reaction (c) ?

ASSIGNMENT 16

CHAPTER XII

THE STRUCTURAL SHOP

The departments of an ordinary structural shop in which the student is more or less concerned are:

Estimating or computing department.

Detailing or drafting department.

Templet shop.

Beam shop.

Assembling shop for girders and trusses.

Yard.

72. Estimating Department.—When a steel structure is to be built, the engineer representing the party that is to own the structure prepares rules regarding the loads to be used in the computations, the permissible unit stresses, the quality of the materials, and the character of the workmanship. These rules are called specifications and they cannot be successfully prepared except by an engineer of experience.

After the preparation of the specifications, proposals or bids are invited from structural companies for the manufacture and erection of the structure. The advertisement mentions where specifications can be seen and information obtained, and names the day and hour when the proposals will be opened.

The estimating department of the structural shop computes the stresses and makes a stress sheet, giving the principal dimensions and sections, and from this prepares an estimate of the weight which enables the amount of its bid to be determined. If it secures the contract, the stress sheet is turned over to the drafting department, where the details are worked out and the working drawings are prepared.

73. Drafting Department.—As soon as a building job is put into the drafting room, the first thing the draftsman generally does is to find out what material is required and how much of it. If the structure is of considerable size, this is best done by laying out the work in a general way on thick brown paper prepared for

this purpose, not stopping to put in the details but going far enough to enable him to determine quite closely the lengths and sizes of the angles and plates. He then consults the list of material in stock, and, if he finds any in his bill that is not in stock, he makes out a *mill bill,* from which the material is ordered immediately from the mill by the purchasing department, for it must be on hand as soon as the drawings are finished.

Shop drawings are then made. In shops of considerable size and if the amount of contract warrants, several men, constituting what may be called a squad, are placed under a checker or squad leader, who is said to have charge of the job in question.

After the drawings have been completed *shop bills* are prepared. A shop bill is made out for each sheet of details and should include the exact length of all material necessary to manufacture the members shown. A shop bill sometimes appears on the right hand side of the shop drawing. When written on special forms it should be so written that each sheet can have its own bill attached if so desired. One page of shop bills should not contain bills for two sheets of drawings. The shop bills serve as a guide for laying out and assembling the work, and one should strive to so group the items as to facilitate these operations. When shipping lists are made from the shop bills, the shipping mark of every separate assembled piece should be given, and usually for identification, the material and length of the principal shapes.

A separate bill is made of all pieces which require forging, such as eye-bars, ties, and counters. Full dimensions and details, and perhaps sketches, are required on this sheet, which then goes to the forge shop.

After the shop drawings are made, checked, and approved, they are blue-printed and sent into the templet shop.

74. Templet Shop.—The templets for much of the work, especially that requiring great nicety of fit, are laid out in full size on the floor of the shop, while for small and general work each templet is made separately from the detailed shop drawings. A templet is either a board or framework of boards, the plane of one of its sides being an exact representation of the plane of one of the sides of the metal shape or piece which is to be made. Holes are bored in the wood at all points where rivets are to be driven. This templet is then clamped upon the piece, and the positions of the holes, bevels, and notches are marked upon the metal face.

75. Beam Shop.—The work on beams and channels is carried on in what is called a beam shop. In case the material has been shipped from the mill according to mill bill made out in the drafting room, the beams are found in that portion of the stock yard reserved for the job in question. If, as in the case of a rush job, the material is said to be taken from stock, the proper lengths are cut from stock lengths with a cold saw or beam shear, which is usually located in the stock yard. In the beam shop the templet is laid on the beam and the positions of the holes, bevels, and notches, are marked upon the metal. The centers of the holes are marked by striking with a hammer a center punch which snugly fits the holes in the templet, the position of each hole being thus indicated by a small indentation in the metal. The bevel and notch lines are drawn on the metal by scratching or marking with chalk along the edges of the templet.

The beam is then taken to the power punches. One line of punches is devoted to punching the webs, another line the flanges. The beam is next taken to the coping machine and coped. If there is any special cutting or bending to be done, it is performed before the piece is taken to the riveting machine to have connections riveted on. The beam is then taken to the end of the shop nearest the yard and the parts not accessible after assembling are thoroughly painted.

76. Assembling Shop.—In this shop all the material which goes to make up the various members of the structure is brought together, fitted, riveted, and made of exact dimensions; when the members leave this shop, they are ready to be prepared for shipment. The order of operations is much the same as in the beam shop. Pneumatic portable riveters are used on the heavier pieces of work. For columns there is an additional operation of planing or milling the ends of the riveted sections before the cap or base plates are riveted on.

77. Yard.—After the material is finished in the shops, it is taken out into the yard and placed in that part of the yard assigned to the particular job number. It should be inspected carefully in order to see that it conforms in every way to the specifications and to the drawings. All work should be painted and have the approval of the company's inspector before being shipped.

CHAPTER XIII

SHOP DRAWING

Fig. 139 shows a detailed working drawing of the lower part of a channel column for a building. A splice connection occurs just above the first floor. The horizontal bearing plate at this splice is used to distribute the load from the upper section over the lower,

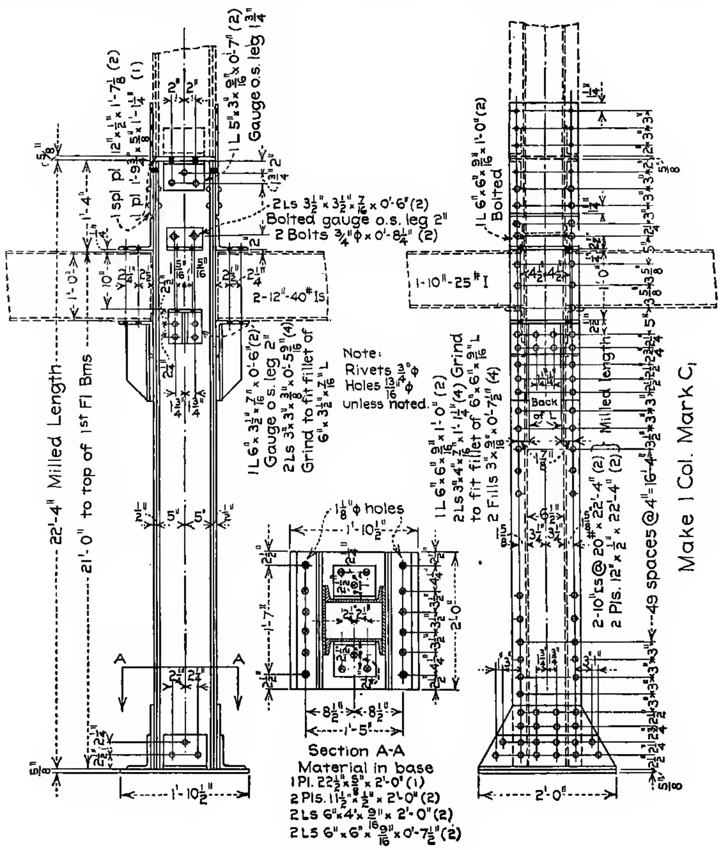


FIG. 139.

both the upper and lower sections being planed or milled to give perfect bearings. The splice plates at the side are intended to hold the two sections in line and to give stiffness to the joint. At the first floor level the floor beam in one direction is a 10-in.

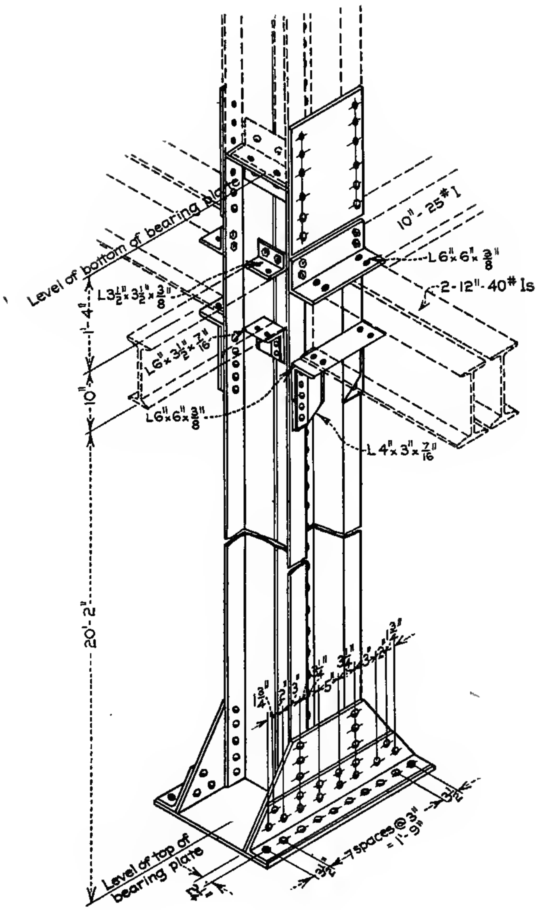

10"-25# I
L6"x6"x 3/8"
2-12"-40# Is
Level of bottom of bearing plate
L3½"x3½"x3/8"
L6"x3½"x7/16"
L6"x6"x3/8"
L4"x3"x7/16"
1'-4"
10"
20'-2"
7 spaces @ 3" = 1'-9"
Level of top of bearing plate

FIG. 140a.

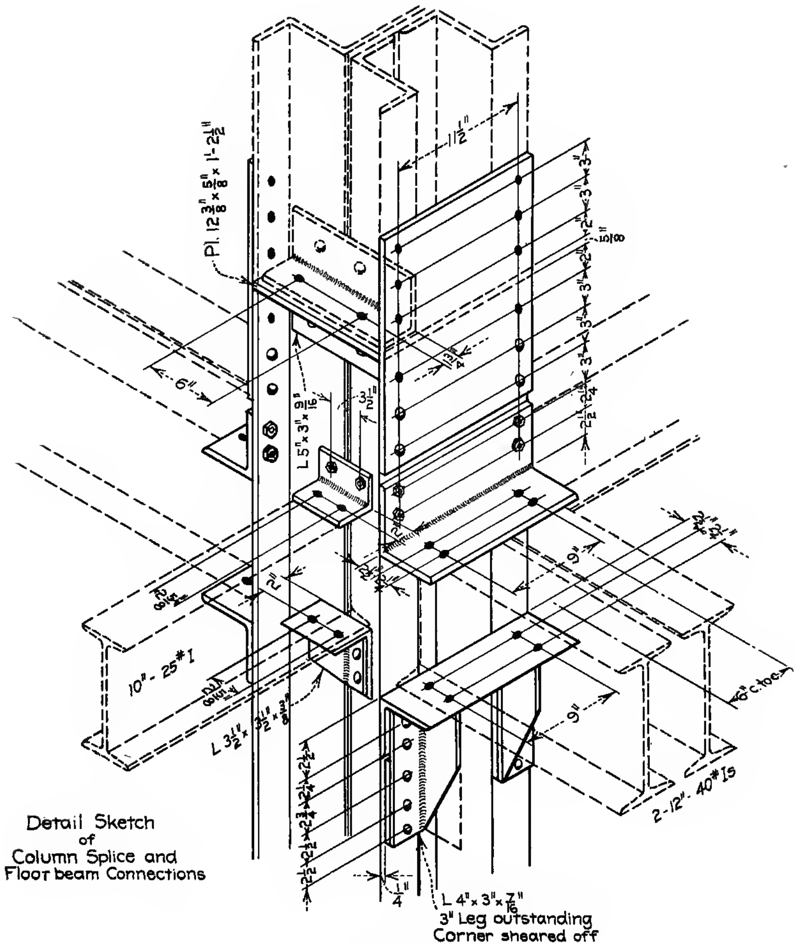

Detail Sketch
of
Column Splice and
Floor beam Connections
Pl. 12 3/8" x 5/8" x 1'-2 1/2"
L 5" x 3" x 9/16"
10"- 25# I
2-12"- 40# Is
L 4" x 3" x 7/16"
3" Leg outstanding
Corner sheared off

Fig. 140b

25-lb. I and in a direction at right angles to this, two 12-in. 40-lb. Is are employed. These are not usually shown on a working drawing but are given here for the benefit of the student. The design of the base of this column is a common one since by such an arrangement the weight is distributed quite uniformly over the foundation. The four blackened holes in the base plate represent bolt holes. Anchor bolts are to be inserted in these holes in order to connect the column firmly to the

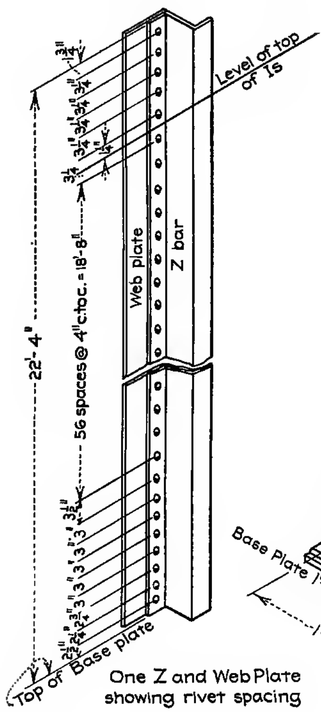


FIG. 140c.

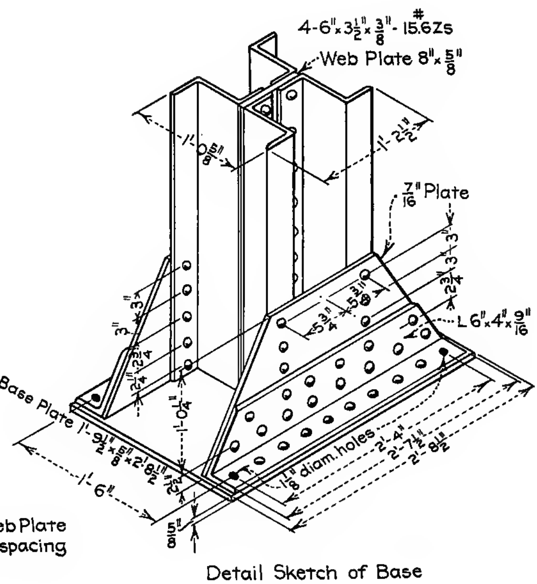


FIG. 140d.

foundation. The lower part of the column is milled before the base plate is riveted on. Bolts are used in place of field rivets in the floor beam connections.

The number in parenthesis following the size of a member is the number of times that exact size occurs. *Gauge* of an angle is the distance from back of angle to line of rivets, see pages 56 to

58, inclusive, in Cambria. The abbreviation *o.s.* means *out standing.*

For practice in drafting structural shapes and putting them together correctly, the following problem is given:

Problem 120.—Make a working drawing of a Z-bar column, the floor beams and floor beam connections to be practically the same as those in the channel bar column of Fig. 139. Follow carefully the dimensions and rivet spacing shown in Fig. 140a, 140b, 140c, and 140d. The student will need to compute the lengths of the separate members. Allow 1/4 in. for clearance above the I-beams, so that the holes in the connection angles just above these beams will surely match the holes in the column, when the work is put togather in the field. The floor beams are shown dotted to gain clearness. Scale to be 1″ = 1′-0″.

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